Identification of Linear Systems in the Presence of Nonlinear Distortions

A Frequency Domain Approach

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Todays lesson

Frequency Response Function Measurements (February 8)

Impact of Nonlinear Distortions on the Linear Framework (February 15)

System Identification (February 22)

Identification of Linear Systems (March 1)

Identification of Nonlinear Systems (March 8)

Outline

Introduction

Approximation of nonlinear systems: important aspects

A nonlinear framework

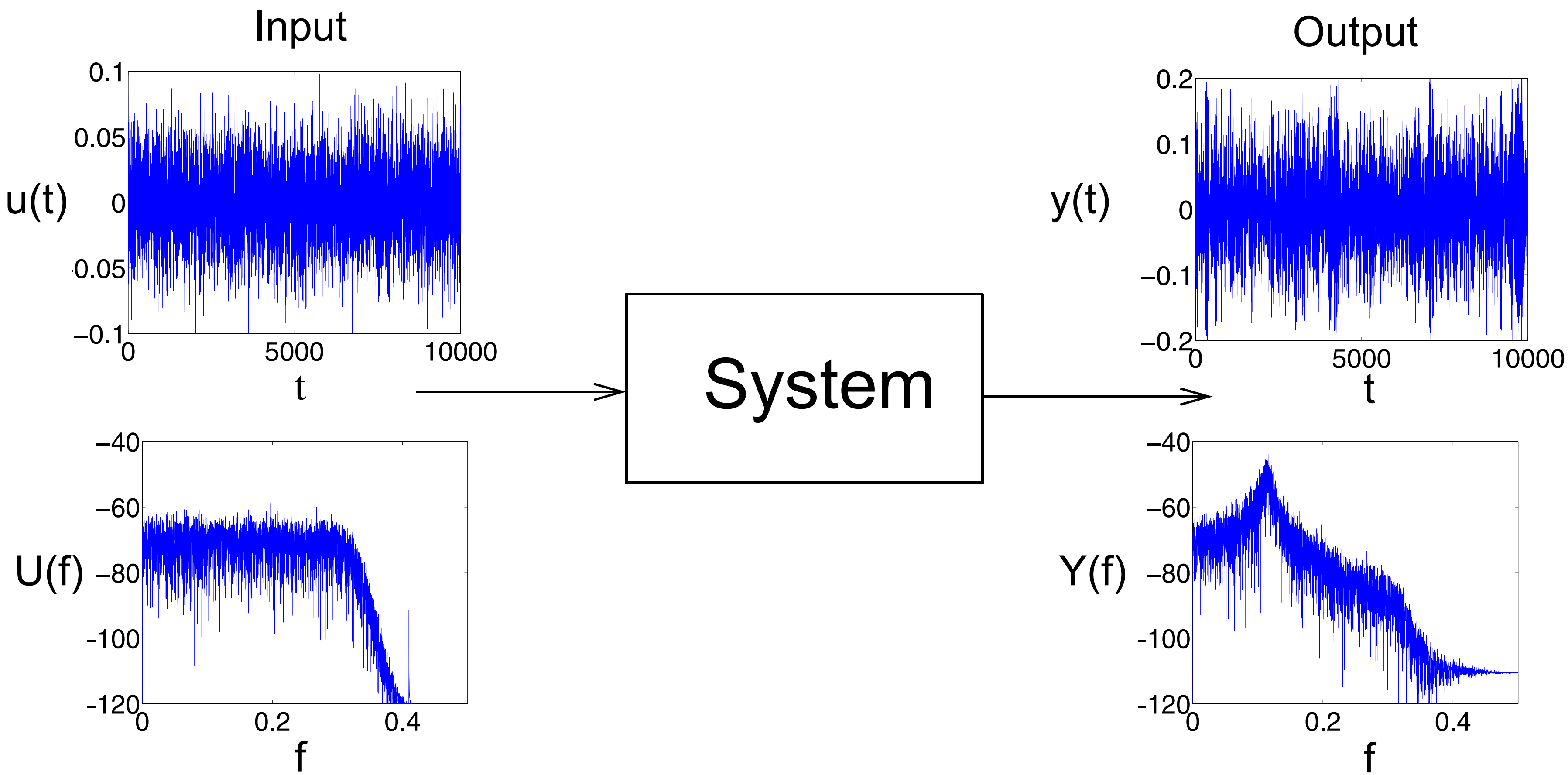
Coherent and non coherent output contributions

A new paradigm

Detection, qualification, quantification of nonlinear distortions

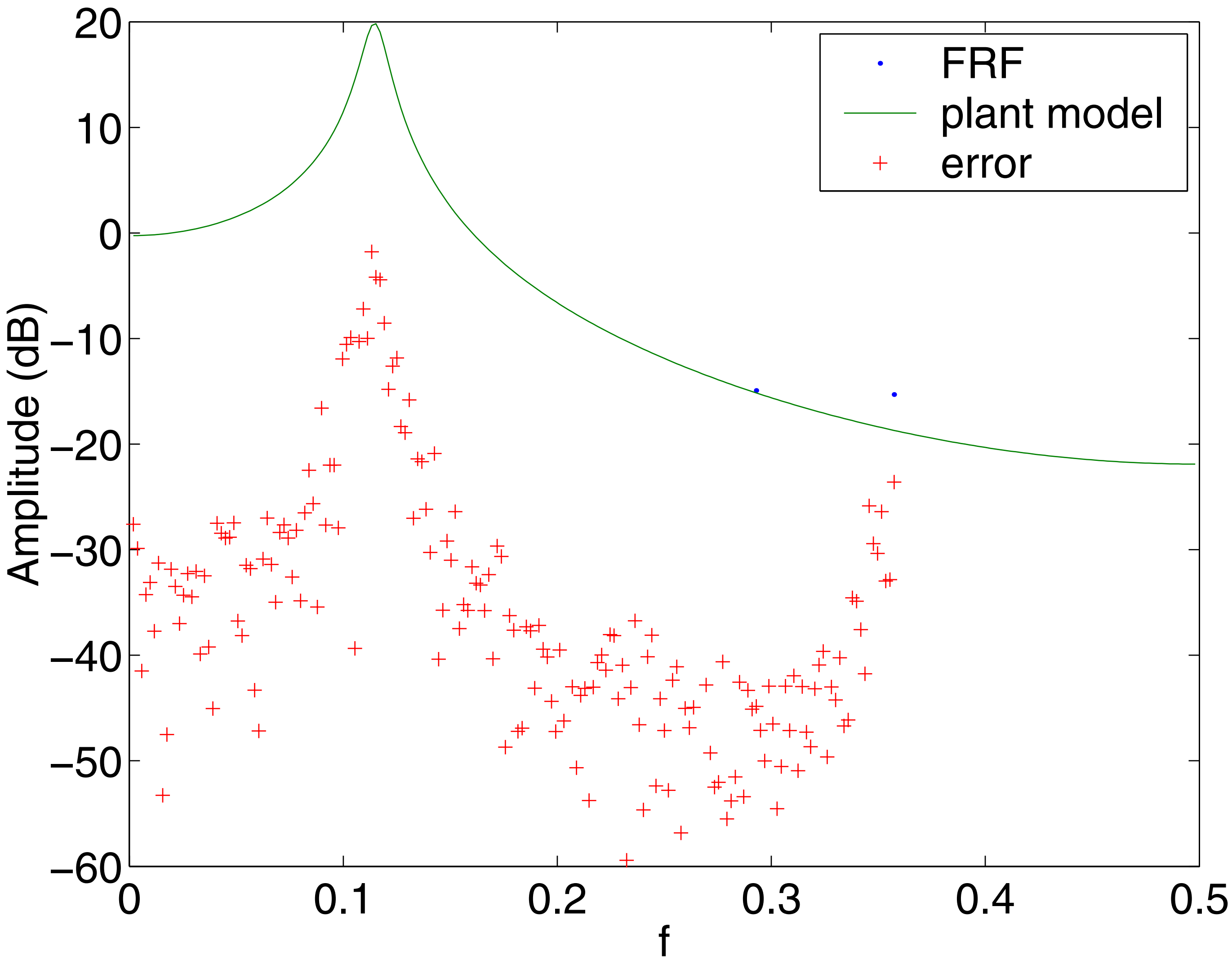
Conclusions

Example

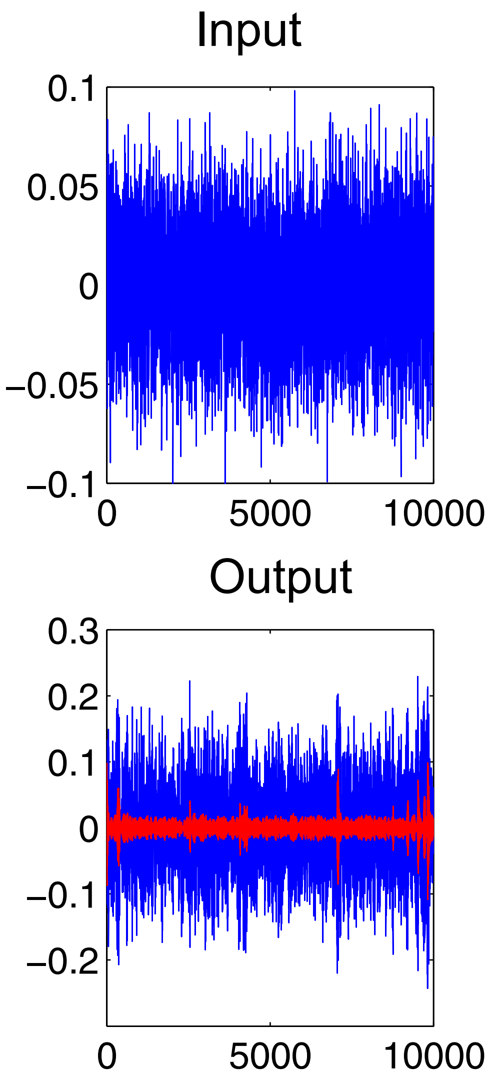


Input
u(t)
0.1
0.05
0
-0.05
-0.1
0
5000
10000
t
U(f)
-40
-60
-80
-100
-120
0
0.2
0.4
f
System
Output
y(t)
0.2
0.1
0
-0.1
-0.2
0
5000
10000
t
Y(f)
-40
-60
-80
-100
-120
0
0.2
0.4
f

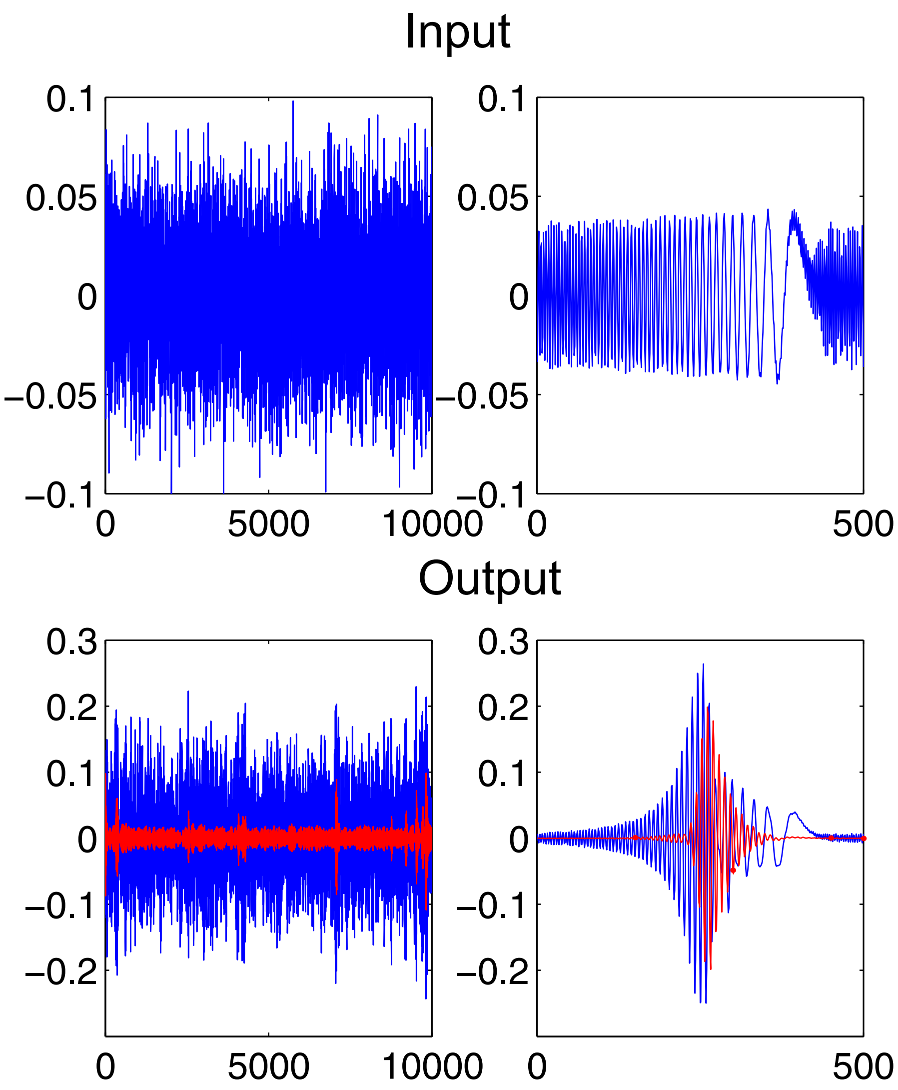
Example
Estimated model



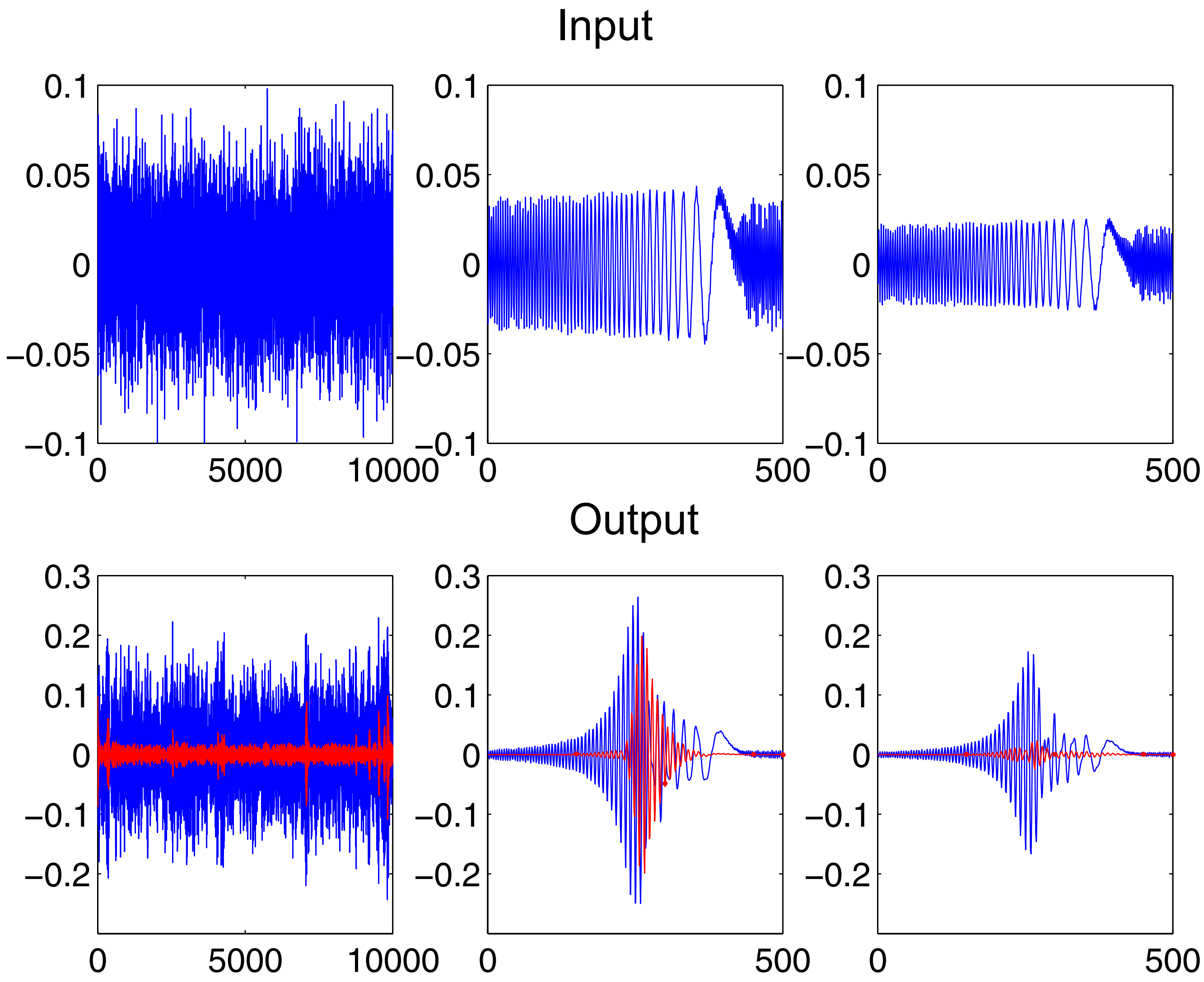
Simulation 1



Validation 2



Validation 3



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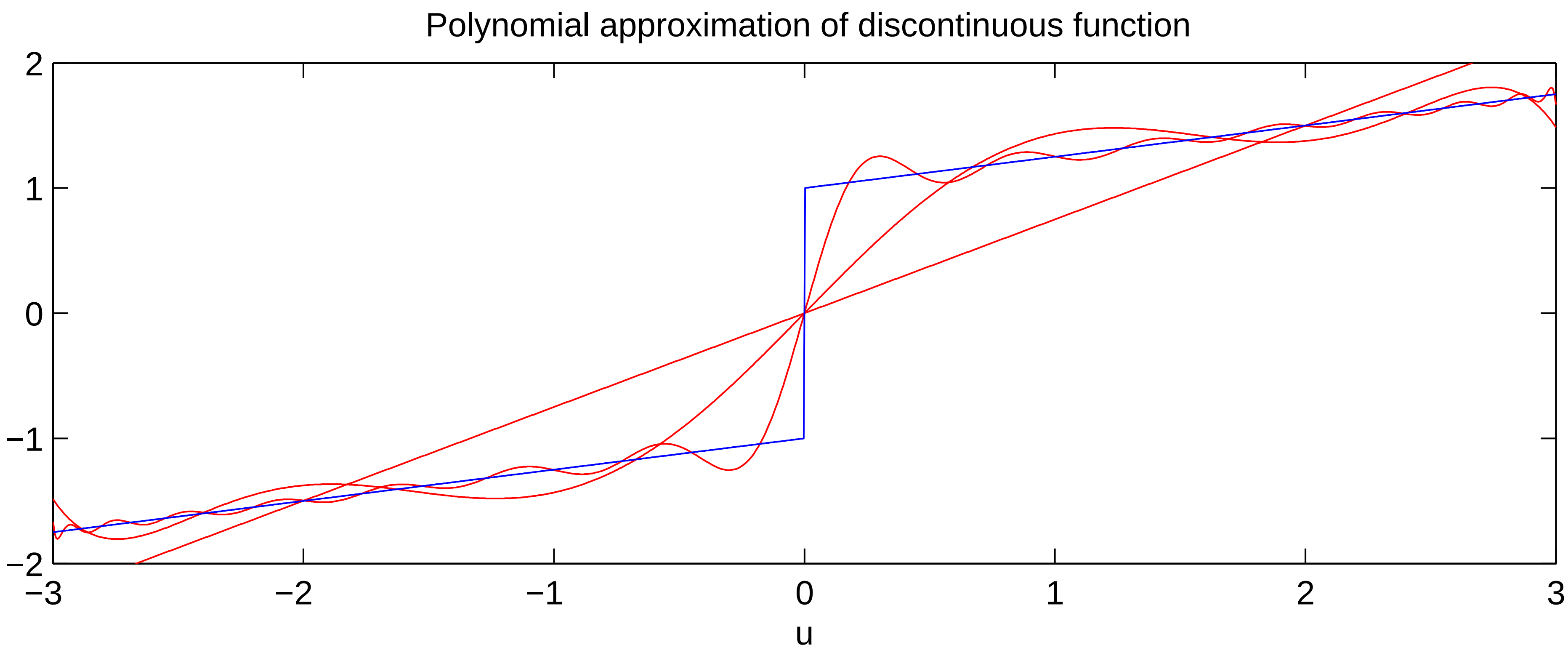
Approximation of nonlinear systems

important aspects

- convergence criterion
- approximation method
- excitation

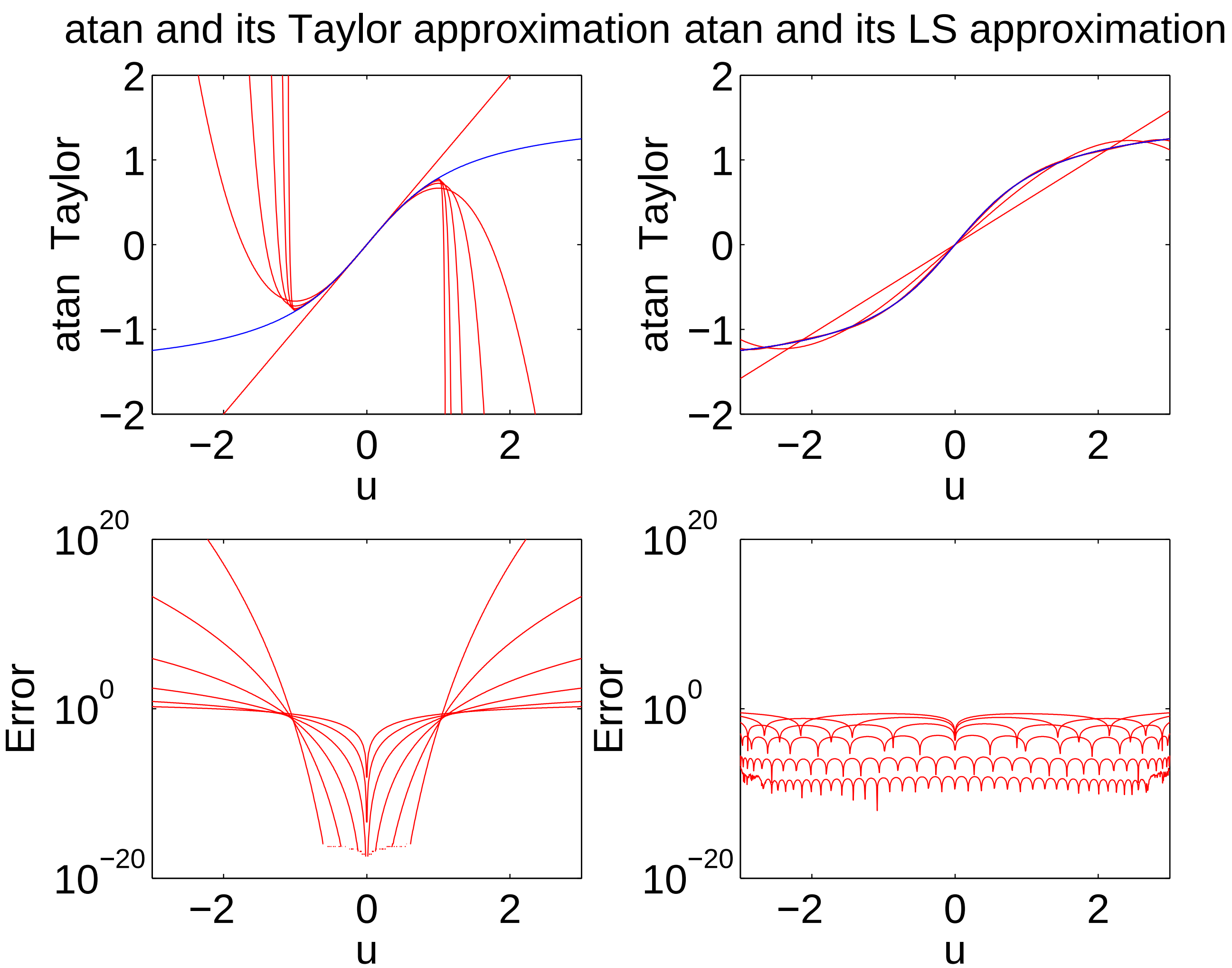
Approximation of nonlinear systems

convergence criterion



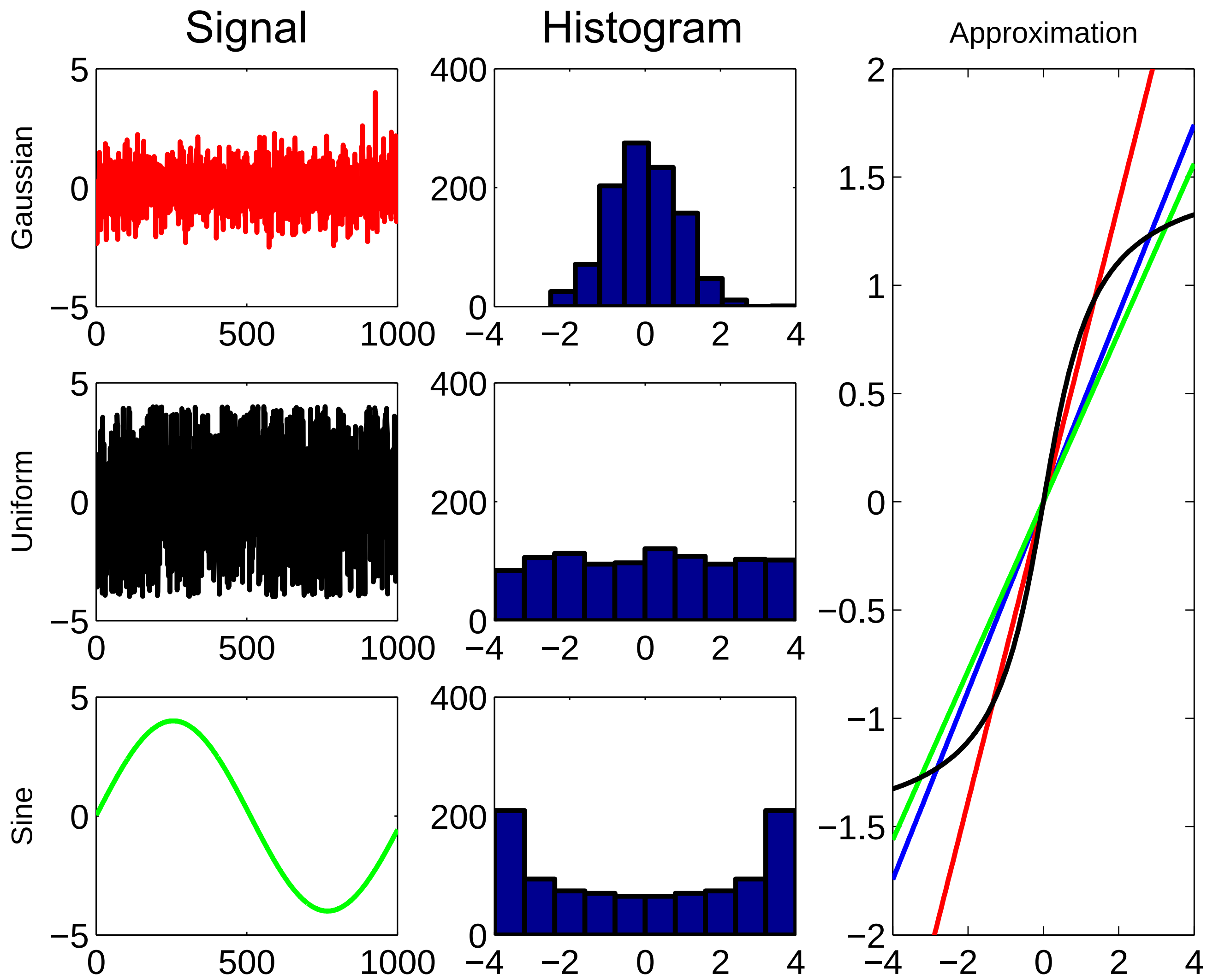
uniform convergence >< point wise convergence

Approximation of nonlinear systems
Approximation method

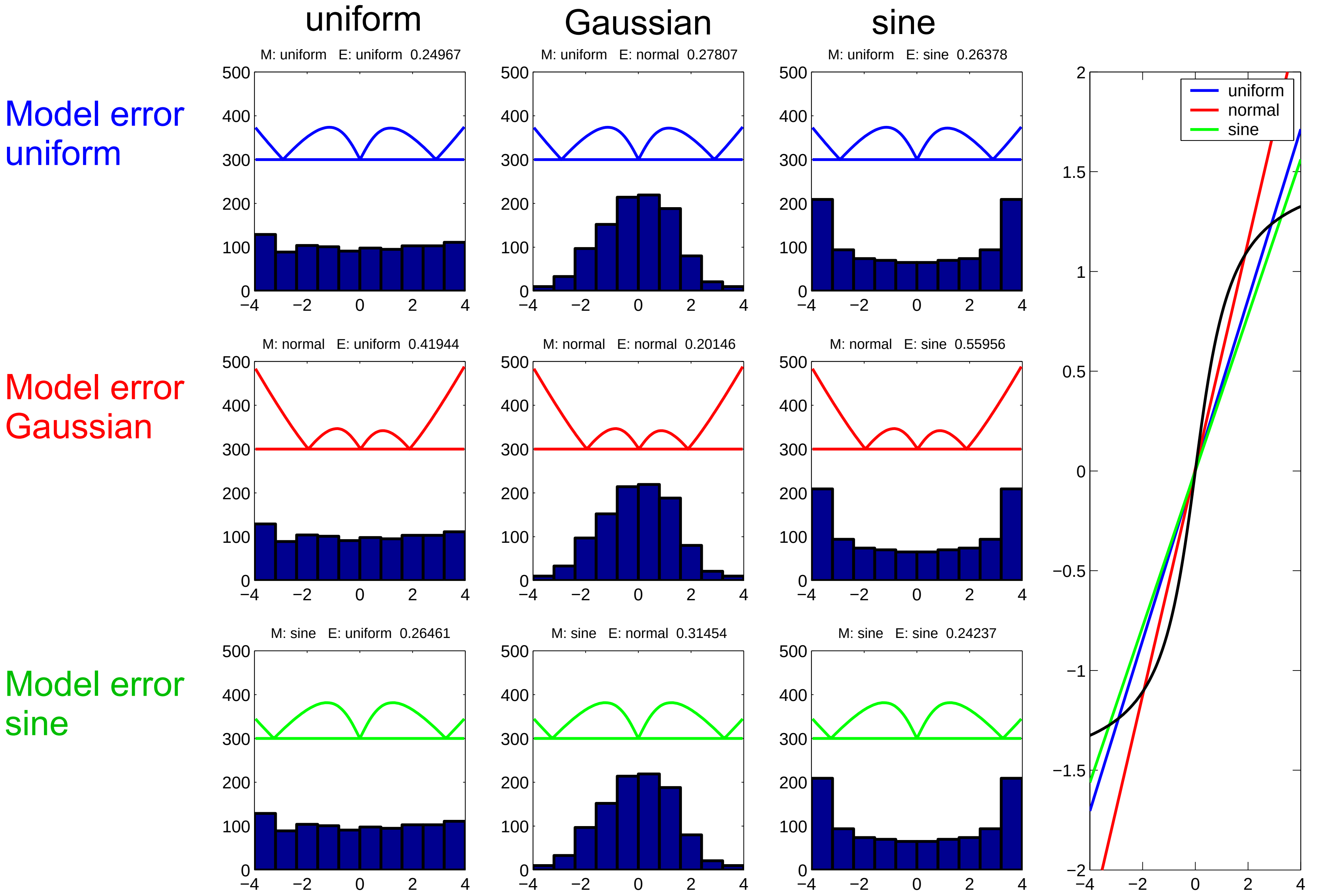


Taylor >< Least Squares

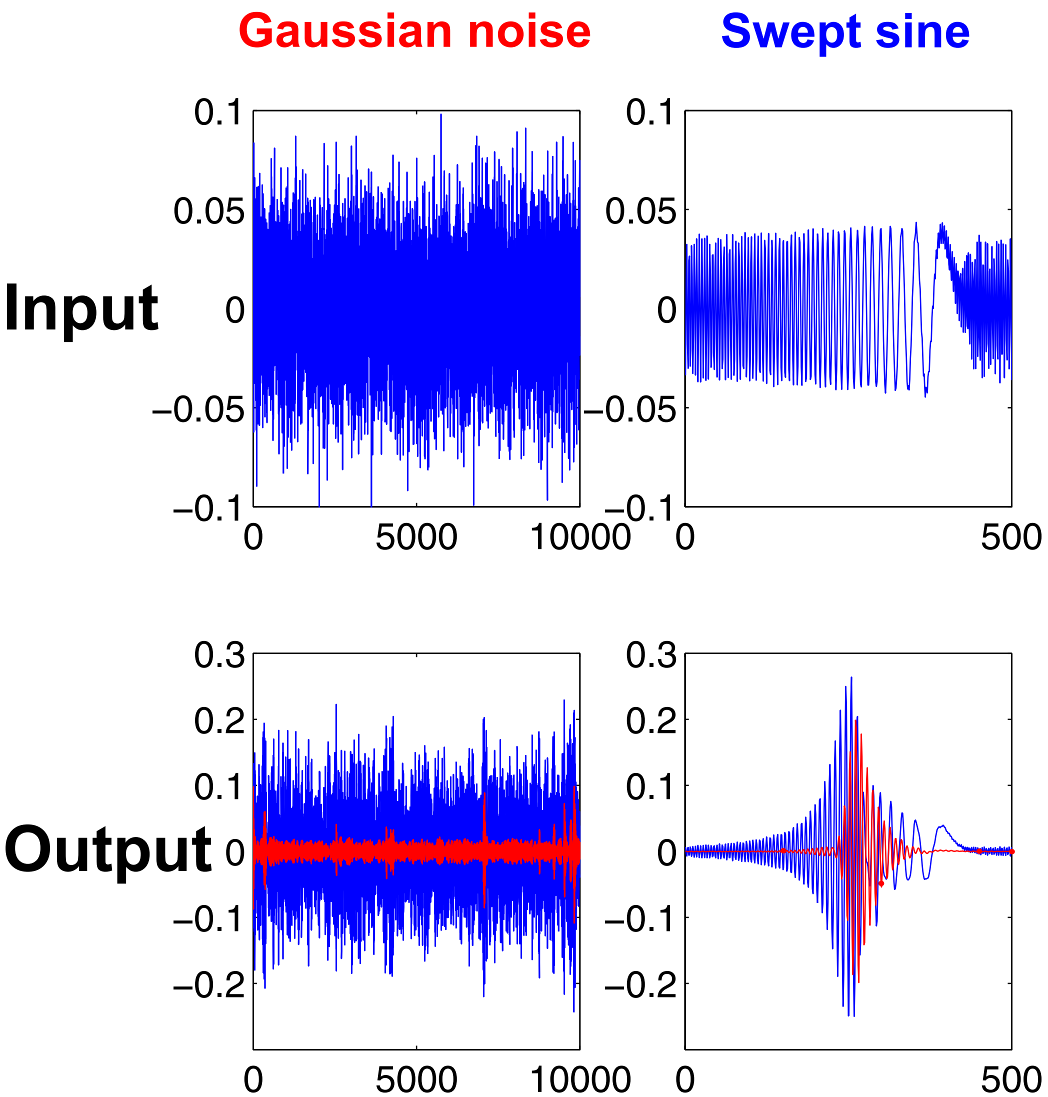
Approximation of nonlinear systems
Excitation

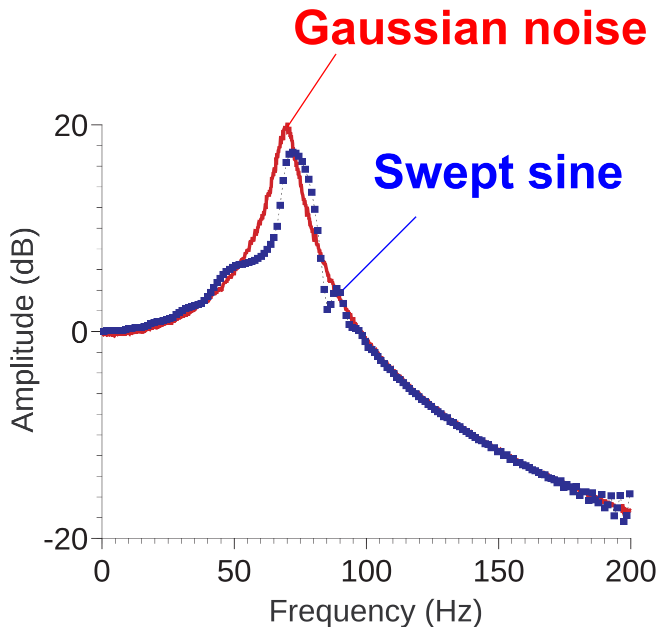


Approximation of nonlinear systems
Excitation (Cont'd)



Approximation of nonlinear systems
Excitation (Cont'd)





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Identification of linear systems with nonlinear distortions

Class of excitations

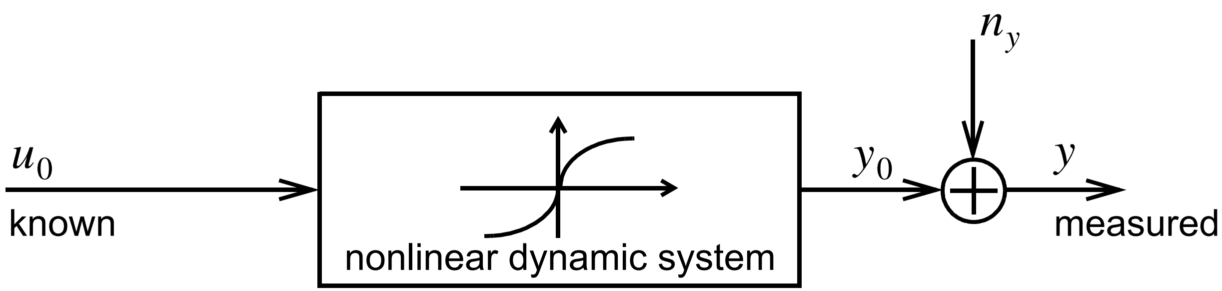
Class of systems

Approximations

Recycling linear identification

Nonlinear identification
--> See lesson 5

System



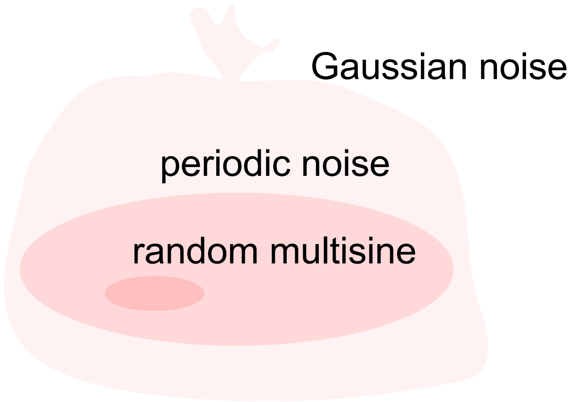
Basic questions

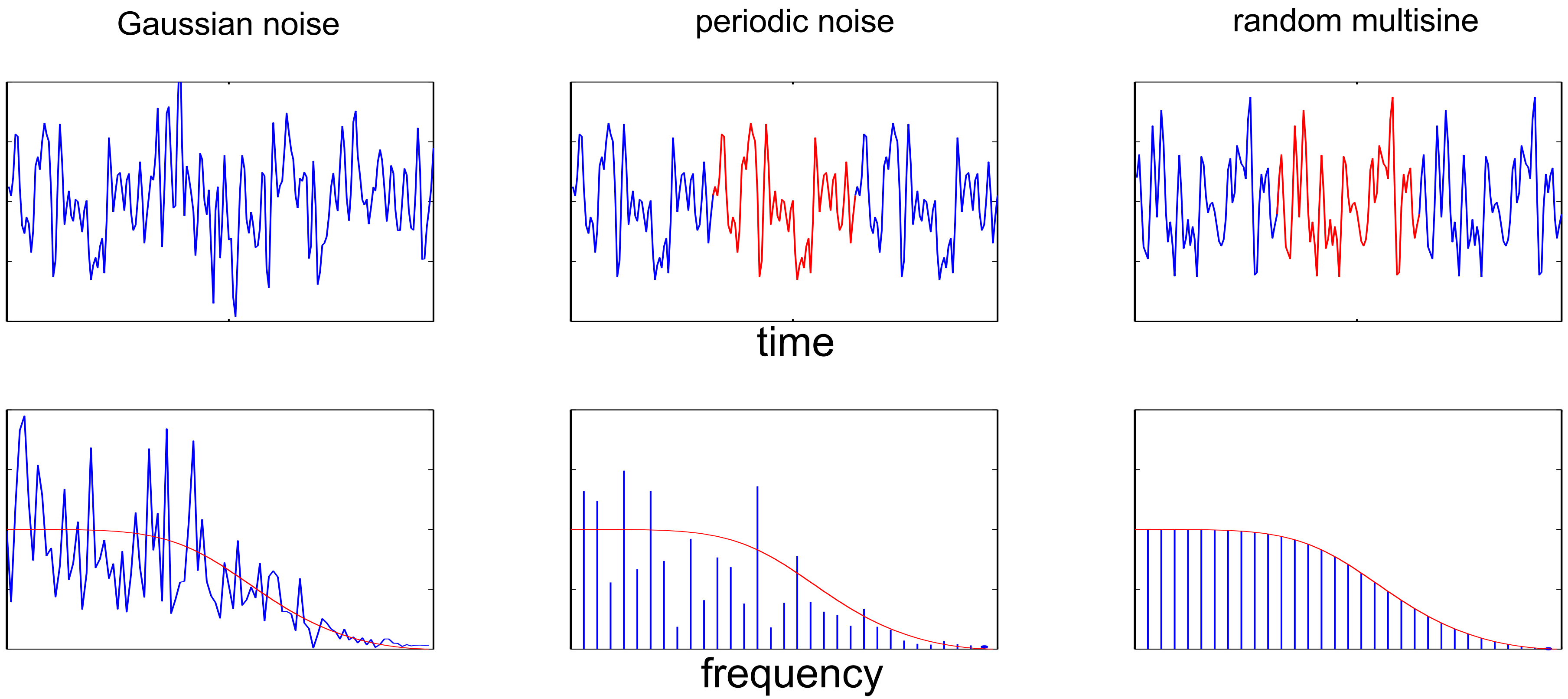
What excitations will be used?

What nonlinear systems are allowed?

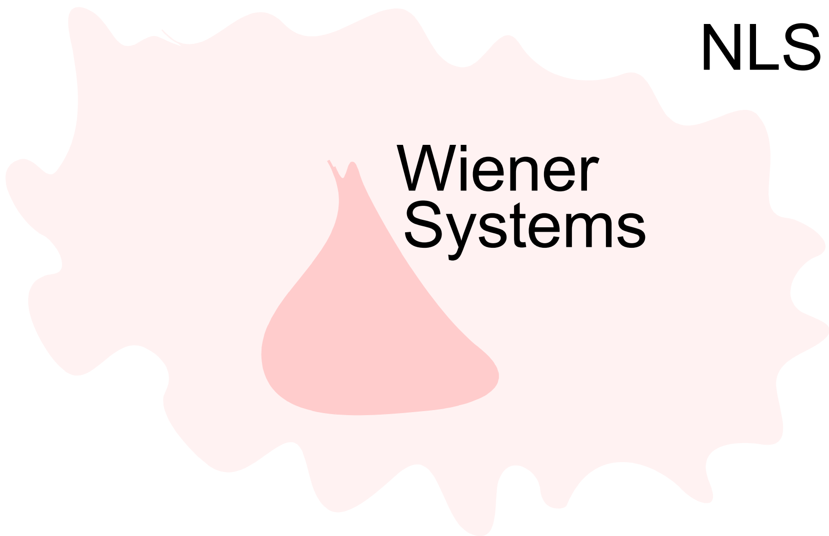
What approximation criterion will be used?

Basic questions: Class of excitation signals

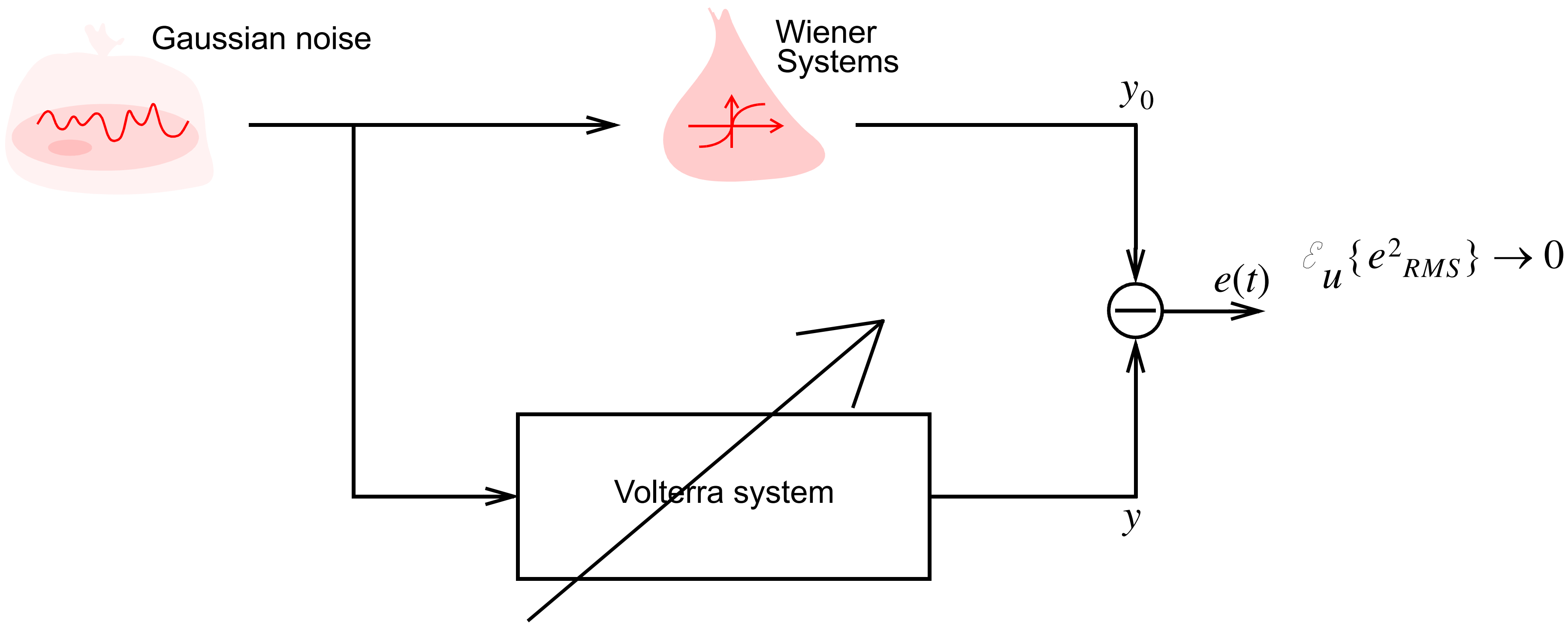




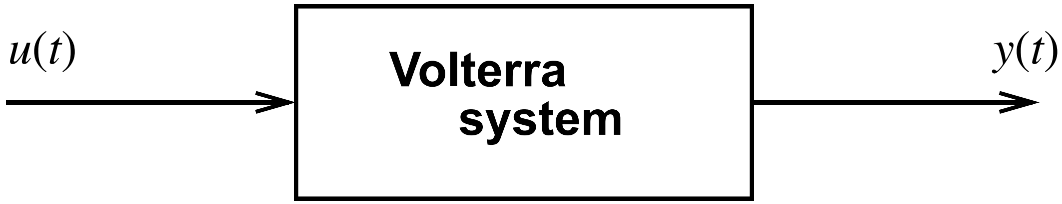
Basic question: What nonlinear systems are allowed?



Wiener systems?



Volterra theory in a nut shell
time domain



$$y(t) = \sum_{k=1}^{\infty} y^{[k]}(t)$$

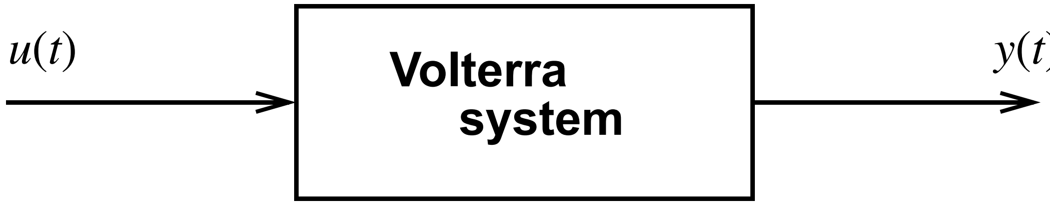
with

$$y^{[1]}(t) = \int_{-\infty}^{\infty} u(\sigma_1) h_1(t - \sigma_1) d\sigma_1$$

$$y^{[2]}(t) = \iint_{-\infty}^{\infty} u(\sigma_1) u(\sigma_2) h_2(t - \sigma_1, t - \sigma_2) d\sigma_1 d\sigma_2$$

...

Volterra theory in a nut shell
multi dimensional frequency domain



$$y(t) = \sum_{k=1}^{\infty} y^{[k]}$$

Define

$$y^{[2]}(t_1, t_2) = \iint_{-\infty}^{\infty} u(\sigma_1)u(\sigma_2)h_2(t_1 - \sigma_1, t_2 - \sigma_2)d\sigma_1 d\sigma_2$$

Then

$$Y^{[2]}(\omega_1, \omega_2) = \iint_{-\infty}^{\infty} y^{[2]}(t_1, t_2)e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}dt_1 dt_2$$

Volterra theory in a nut shell
collapsing the multi dimensional frequency domain

Inverse Fourier transform

$$y^{[2]}(t) = y^{[2]}(t_1, t_2) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} Y^{[2]}(\omega_1, \omega_2) e^{j\omega_1 t_1} e^{j\omega_2 t_2} d\omega_1 d\omega_2 \text{ with } t = t_1 = t_2$$

or

$$y^{[2]}(t) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} Y^{[2]}(\omega_1, \omega_2) e^{j(\omega_1 + \omega_2)t} d\omega_1 d\omega_2$$

Put

$\omega = \omega_1 + \omega_2$ --> $\omega_2 = \omega - \omega_1$, and $d\omega = d\omega_2$

Then

$$y^{[2]}(t) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} Y^{[2]}(\omega_1, \omega - \omega_1) d\omega_1 e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y^{[2]}(\omega) e^{j\omega t} dt$$

with

$$Y^{[2]}(\omega) = \int_{-\infty}^{\infty} y^{[2]}(\omega, \omega - \omega_1) d\omega_1$$

Volterra theory in a nut shell
frequency domain relations

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = H^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) U(\omega_1) \ldots U(\omega_n)$$

with

$$H^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = \int \ldots \int_{-\infty}^{\infty} h_n(t_1, t_2, \ldots, t_n) e^{-j\omega_1 t_1} \ldots e^{-j\omega_n t_{nn}} dt_1 dt_2 \ldots dt_n$$

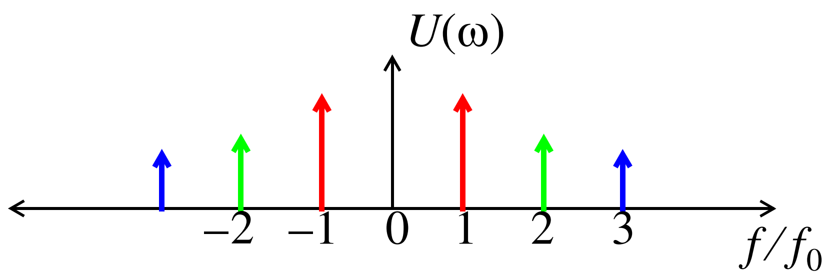
Corresponding one-dimensional frequency representation

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) \to Y(\omega_1 + \omega_2 + \ldots + \omega_n)$$

$\omega_1 + \omega_2 + \ldots + \omega_n$ indicates that contribution results from n^{th} degree NL

Volterra theory in a nut shell
frequency domain relations for periodic signals

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = H(\omega_1, \omega_2, \ldots, \omega_n)U(\omega_1)\ldots U(\omega_n)$$



with

$$Y^{[2]}(\omega) = \int_{-\infty}^{\infty} y^{[2]}(\omega, \omega-\omega_1)d\omega_1 \to Y^{[2]}(k) = \sum_l y^{[2]}(l, k-l) = \sum_l H^{[2]}(l, k-l)U(l)U(k-l)$$

similar

$$Y^{[3]}(k) = \sum_{l_1}\sum_{l_2}\ldots U(l_1)U(l_2)U(k-l_1-l_2)$$

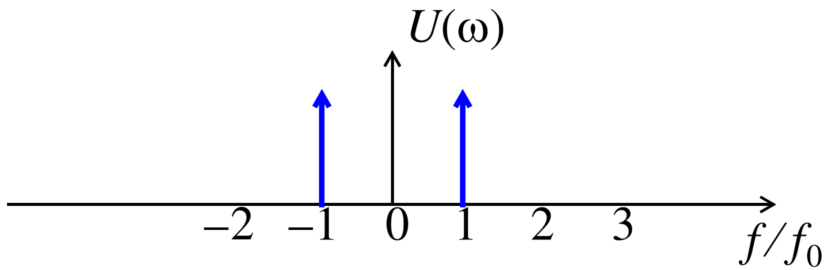
Conclusion

$Y^{[3]}(k)$ sum over all combination $U(l_1)U(l_2)U(l_3)$ such that $l_1 + l_2 + l_3 = k$

Example

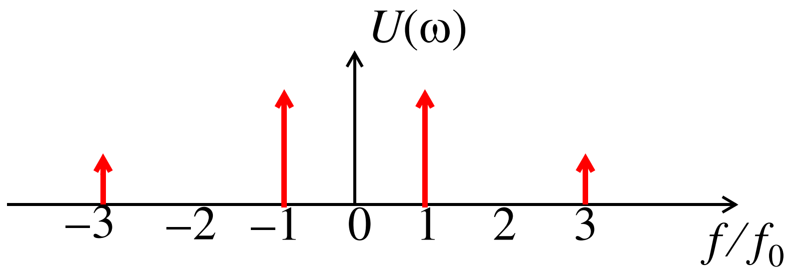
$$u(t) = \sin 2\pi f_0 t = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}$$

$$u^3 = ?$$

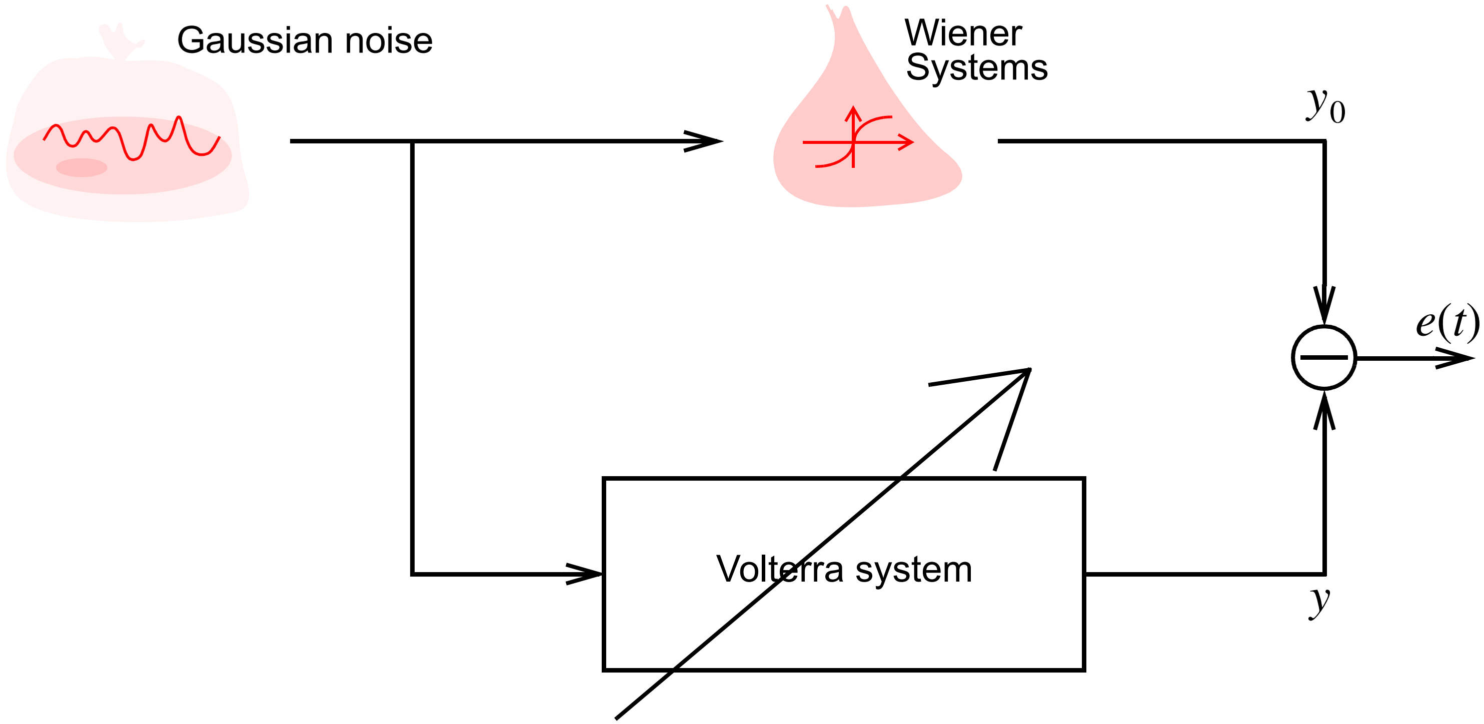


All possible combinations

1	1	1	3
1	1	-1	1
1	-1	1	1
1	-1	-1	-1
-1	1	1	1
-1	1	-1	-1
-1	-1	1	-1
-1	-1	-1	-3

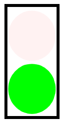
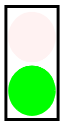
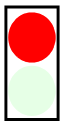


reconsider Wiener systems?



Major properties

- A periodic input $\rightarrow$ a periodic output with the same period

- Approximates the output in mean squares sense
  -  dynamic saturations
  -  discontinuities
  -  bifurcations and chaos

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A nonlinear framework

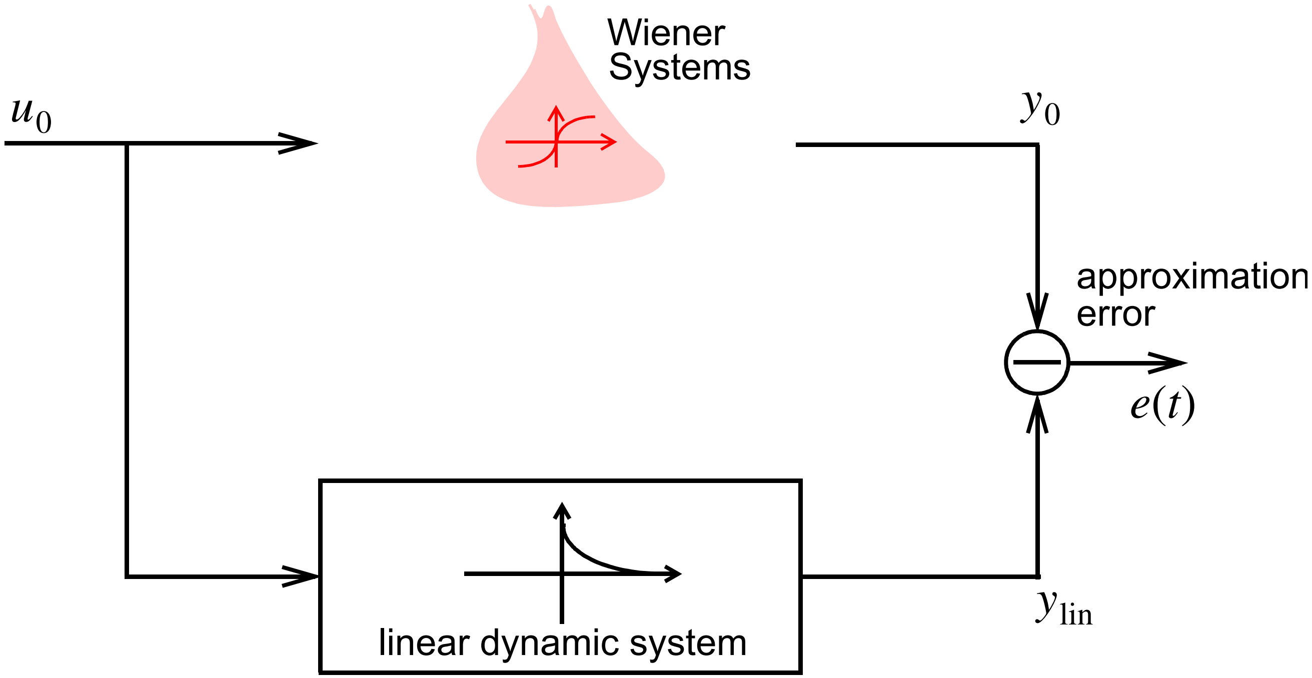
Coherent and non coherent output contributions

A new paradigm

Detection, qualification, quantification of nonlinear distortions

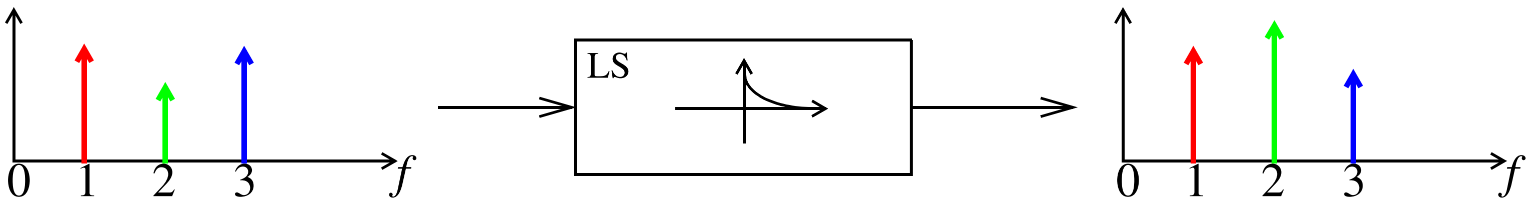
Conclusions

Understanding the impact of nonlinear distortions on the linear framework

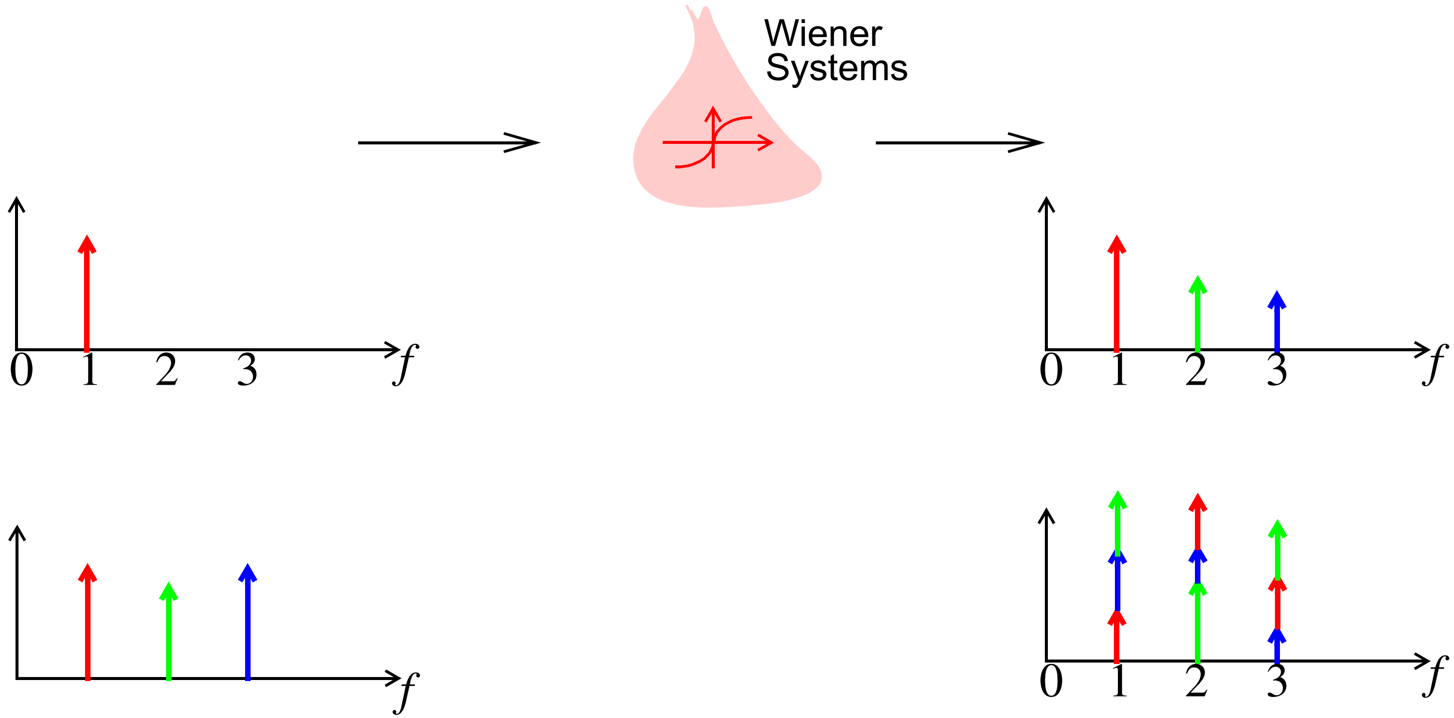


Behaviour of a nonlinear system

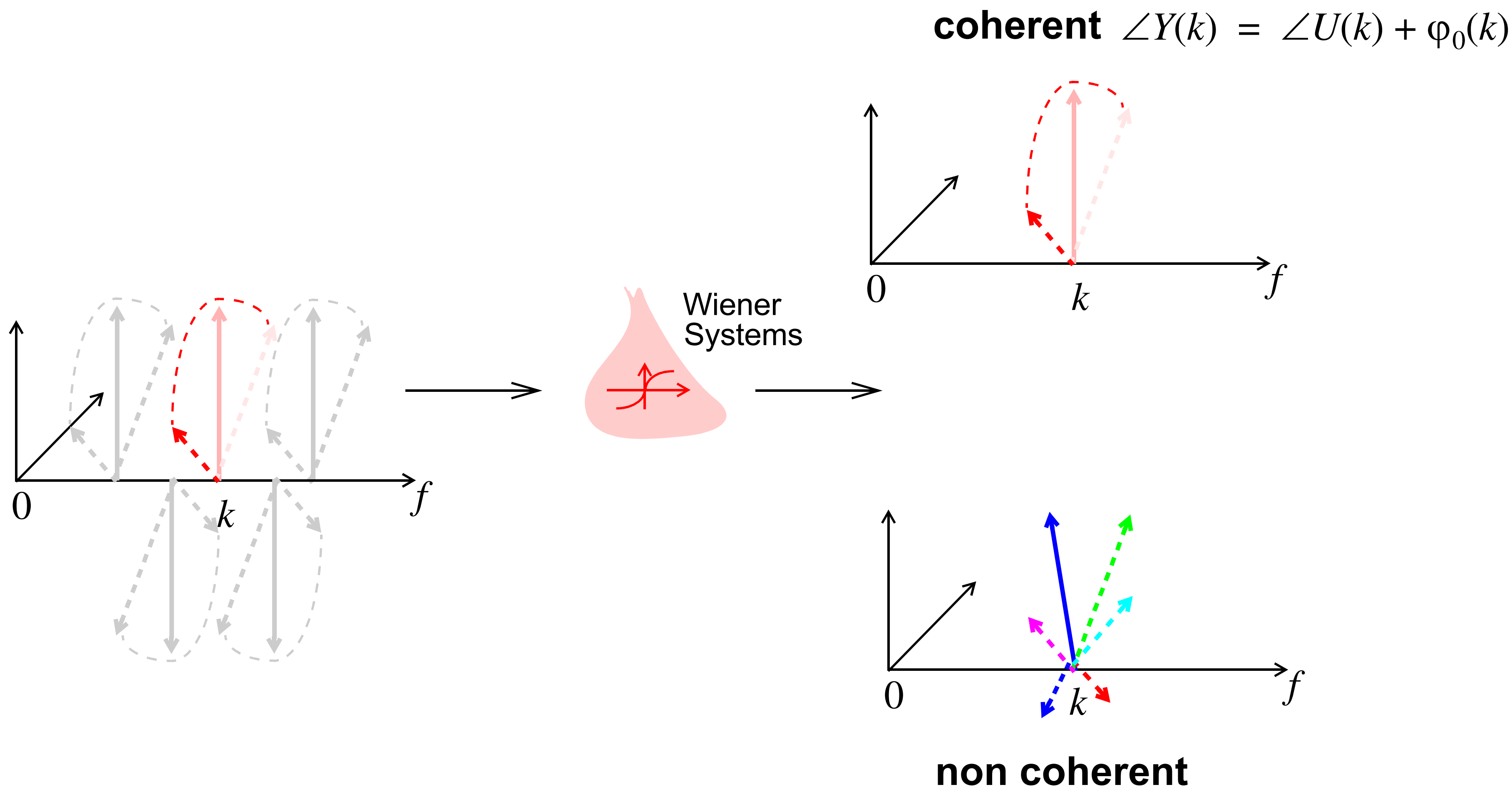
A linear system



A Nonlinear system



Behaviour of a nonlinear system



(non) Coherent contributions

Example: cubic contributions

$$Y^{[3]}(k) = \sum_{l_1}\sum_{l_2} H^{[3]}(l_1, l_2, k-l_1-l_2)U(l_1)U(l_2)U(k-l_1-l_2)$$

Frequency combinations s.t. $\angle U(l_1)U(l_2)U(k-l_1-l_2) = \angle U(k)$?

Yes

$U(k)U(-l)U(l) = U(k)|U(l)|^2$ --> coherent contribution

NO

$U(k-2)U(1)U(1)$ --> non coherent contribution

(non) Coherent contributions (Cont'd)

Example: quadratic contributions

$$Y^{[2]}(k) = \sum_{l_1} H^{[2]}(l_1, k-l_1) U(l_1) U(k-l_1)$$

Frequency combinations s.t. $\angle U(l_1) U(k-l_1) = \angle U(k)$?

Yes

$U(k)U(0)$ --> coherent contribution requires DC

No

$U(k-1)U(1)$ --> non coherent contribution

(non) Coherent contributions

Conclusions

Put $U(0) = 0$

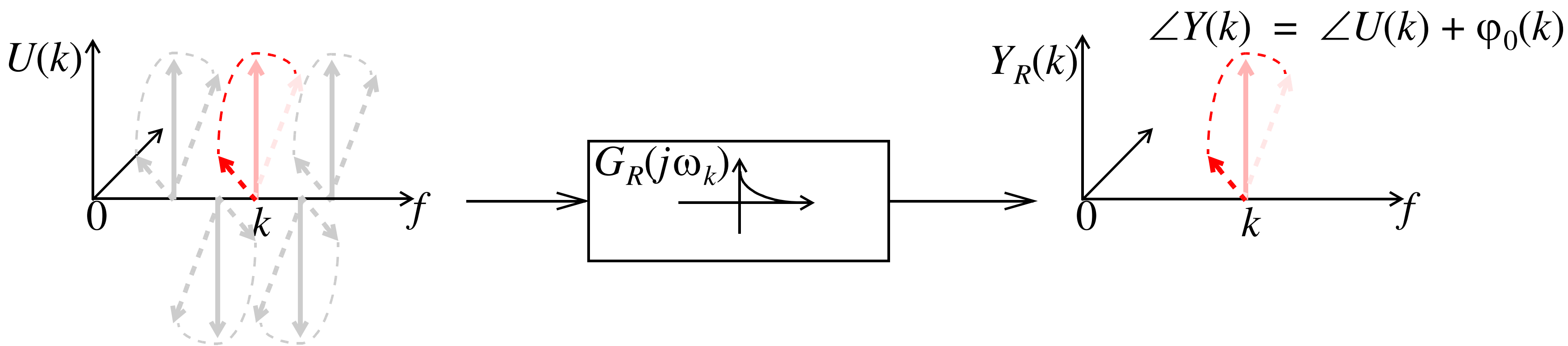
Even nonlinearities

always non coherent

Odd nonlineairities

coherent
+
non coherent contributions

Coherent output

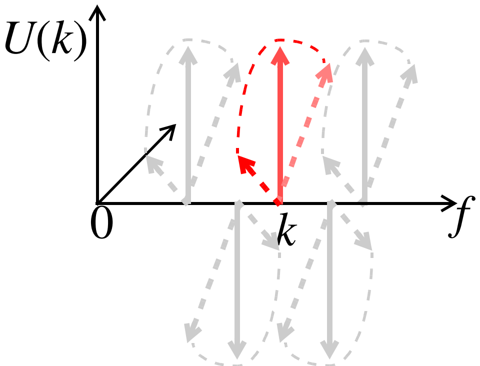


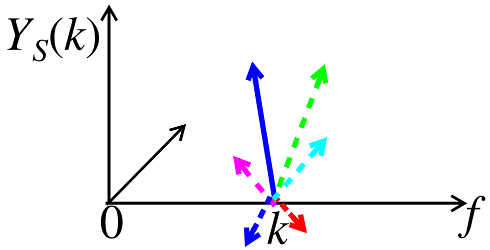
$$Y_R(k) = G_R(j\omega_k)U(k)$$

$G_R(j\omega_k)$ is the best linear approximation

$G_R(j\omega_k)$ is a function of S_{UU}

Non coherent output

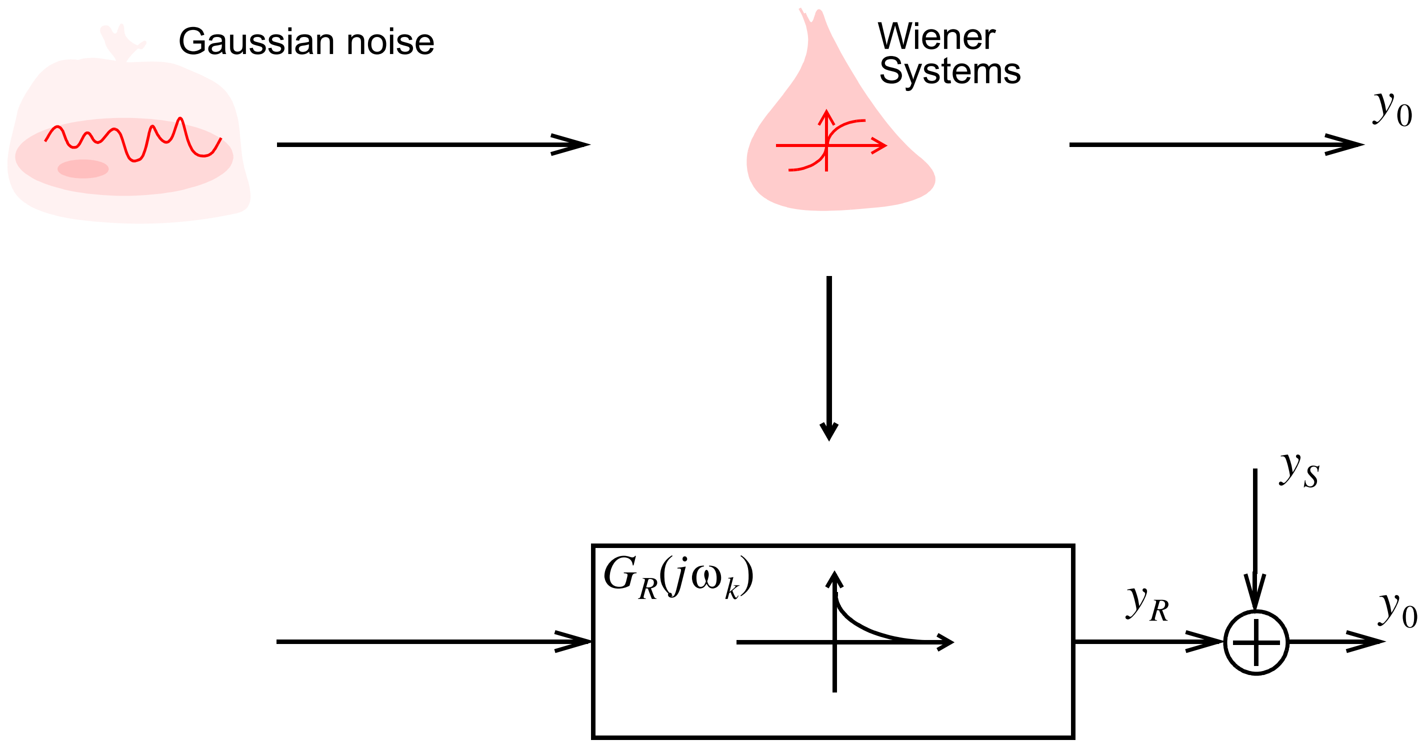




The phase of $Y_S(k)$ depends on $U(l)$ $\quad l \neq k$

$Y_S(k)$ acts as a noise source

A ‘new’ paradigm



$$Y(k) = G_R(j\omega_k)U(k) + Y_S(k)$$

A 'new' paradigm

Properties

$$Y(k) = G_R(j\omega_k)U(k) + Y_S(k)$$

$G_R(j\omega_k)$ is the 'best linear approximation'

- smooth
- $O(N^0)$
- same for all excitations in the set (with same power spectrum)
- only odd nonlinearities contribute

$Y_S(k)$ is the 'nonlinear noise source'

- smooth power spectrum
- zero mean
- circular complex normally distributed
- $O(N^0)$
- same power spectrum for all excitations in the set
- even and odd nonlinearities contribute

zero mean circular complex normally distributed

$$x = a + jb \in C$$

Zero mean circular complex: $E[x^2] \equiv 0$

$$E[(a+jb)^2] = E[a^2 - b^2 + 2jab] = 0 \quad \Rightarrow \quad E[a^2] = E[b^2] = \sigma^2 \text{ and } E[ab] = 0$$

Zero mean circular complex normally distributed: $E[x^n] \equiv 0$

x is a complex vector

- without a preferred direction
- no relation between amplitude and phase

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A new paradigm: some examples

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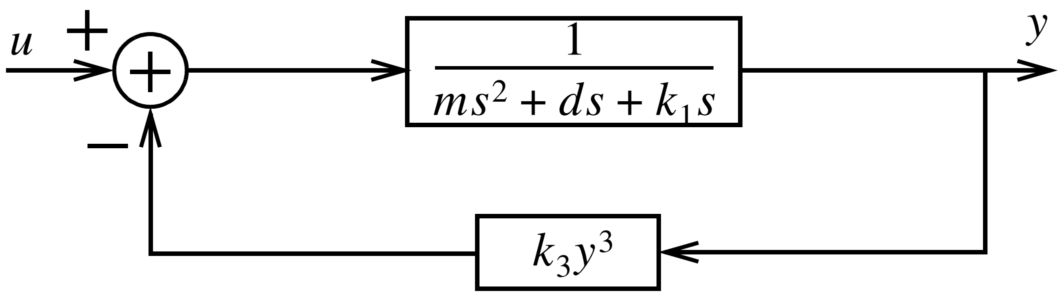
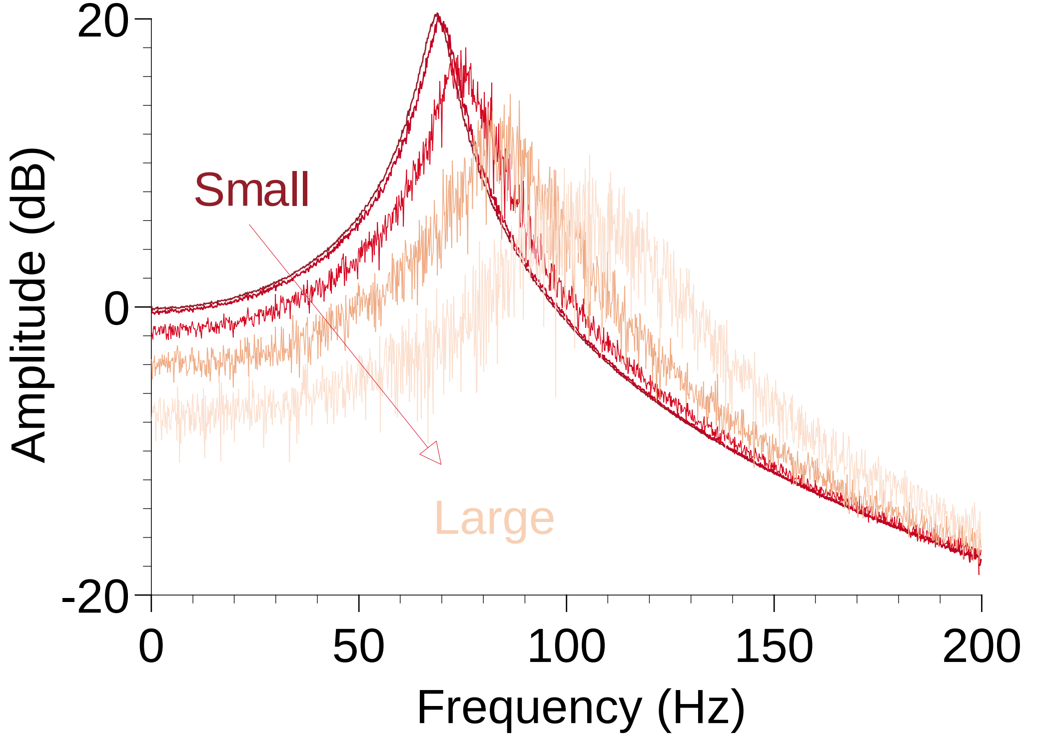
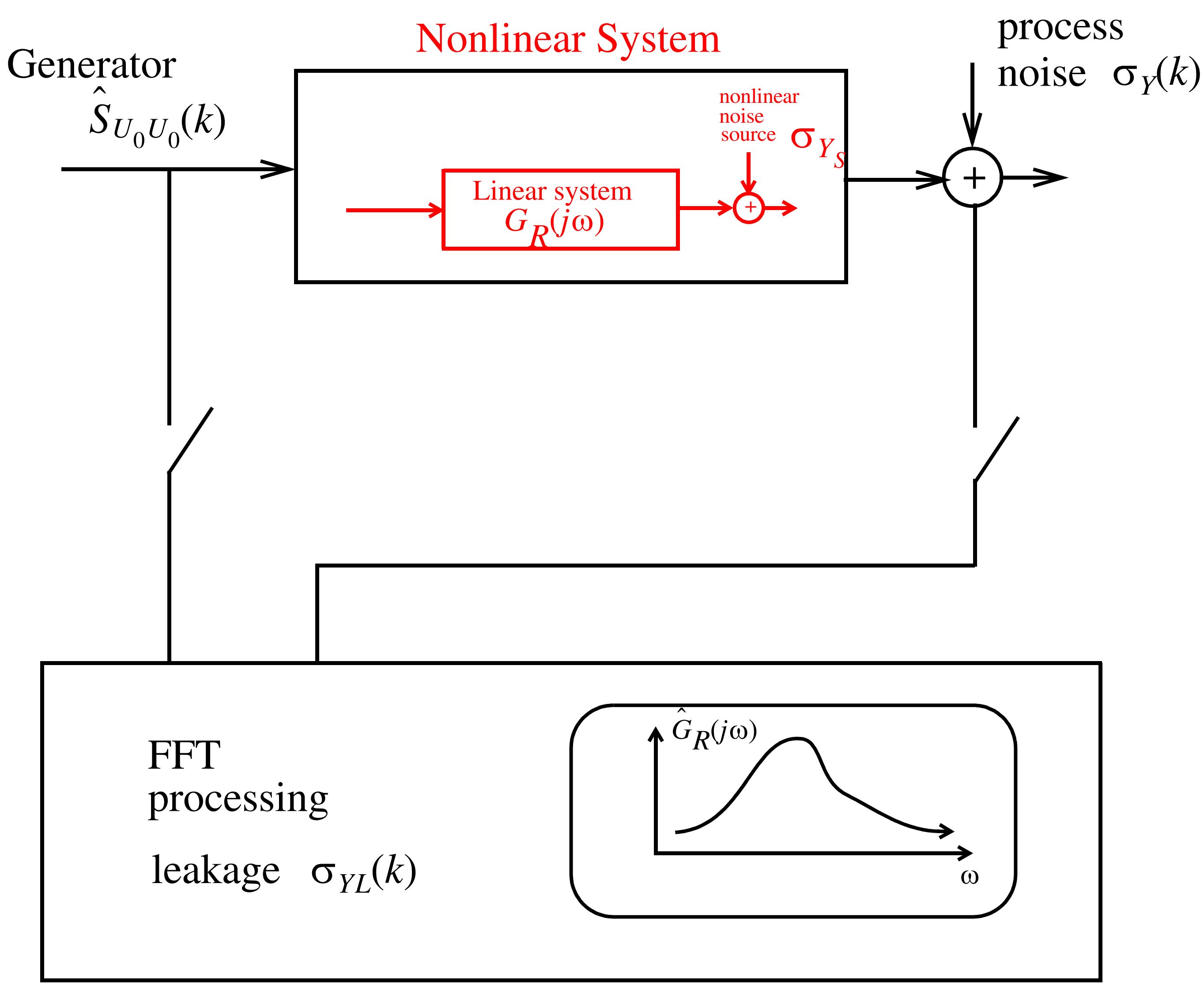
Example FRF measurement of a system with a hardening spring





FRF-measurements in the presence of NL-distortions



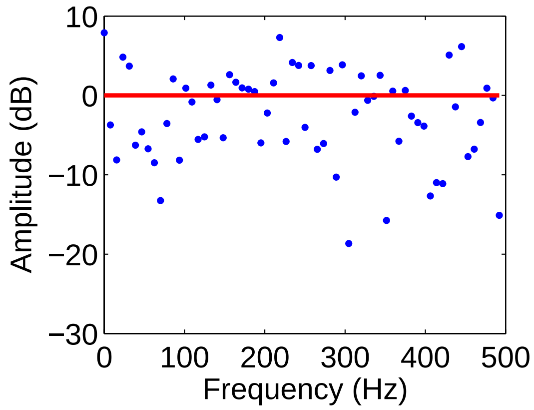
$$\sigma^2_{\hat{G}_R}(k) = |G_R(\omega_k)|^2 \frac{\sigma^2_{YL}(k) + \sigma^2_{Y_S}(k) + \sigma^2_Y(k)}{\hat{S}_{Y_0Y_0}(k)}$$

FRF-measurements in the presence of NL-distortions (Cont'd)

$$\sigma^2_{\hat{G}_R}(k) = |G_R(\omega_k)|^2 \frac{\sigma^2_{\hat{Y}L}(k) + \sigma^2_{\hat{Y}_S}(k) + \sigma^2_{\hat{Y}}(k)}{\hat{S}_{Y_0Y_0}(k)}$$

Avoid dips in $\hat{S}_{Y_0Y_0}(k) = |G_R(\omega_k)|^2 \hat{S}_{U_0U_0}(k)$

deterministic signals >< noise



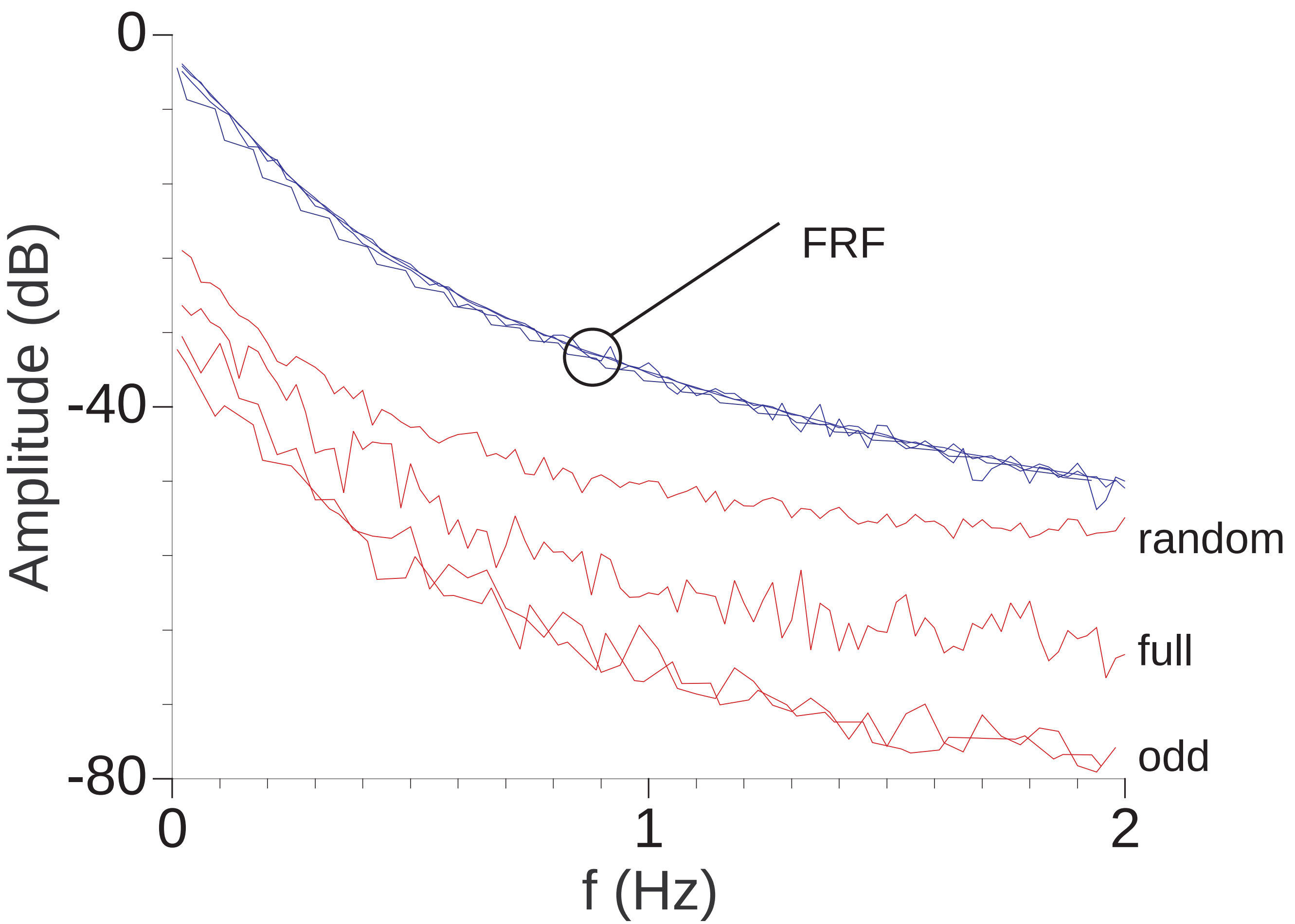
Reduction of the leakage errors $\sigma^2_{\hat{Y}L}$

periodic signals

Reduction of the impact of nonlinear distortions $\sigma^2_{\hat{Y}_S}$

excite only the odd harmonics
or
excitations with an odd amplitude distribution

Example 2: Hair dryer



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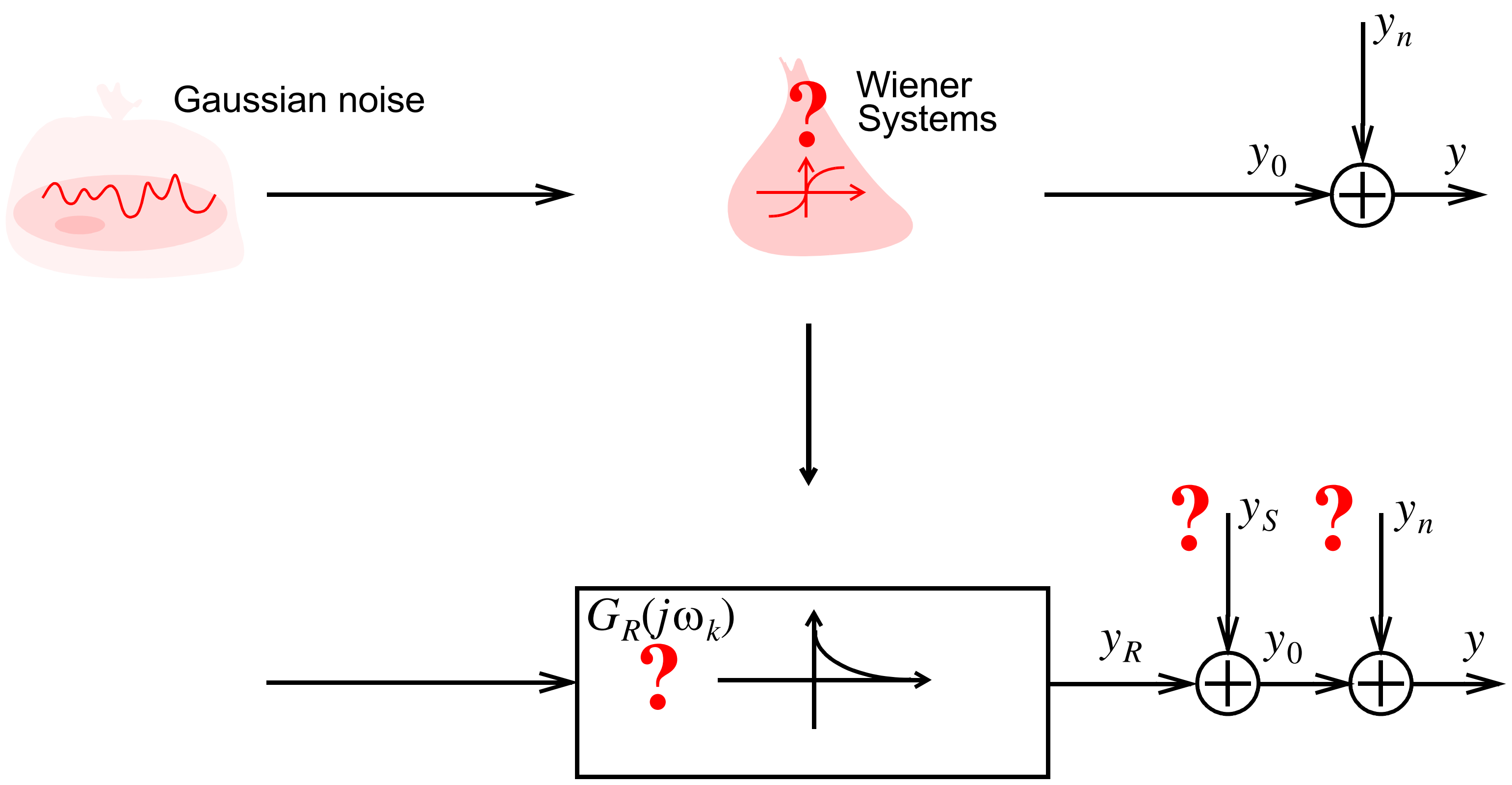
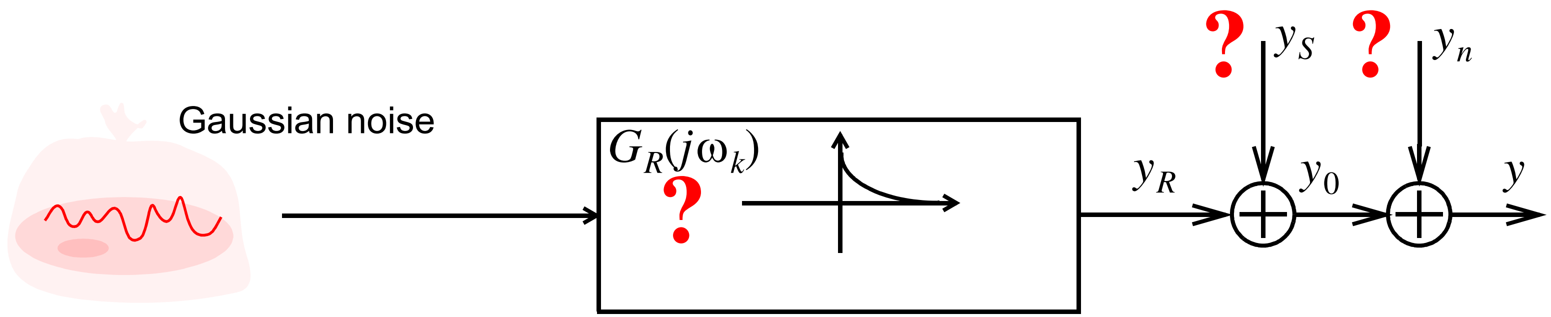
Non parametric modelling

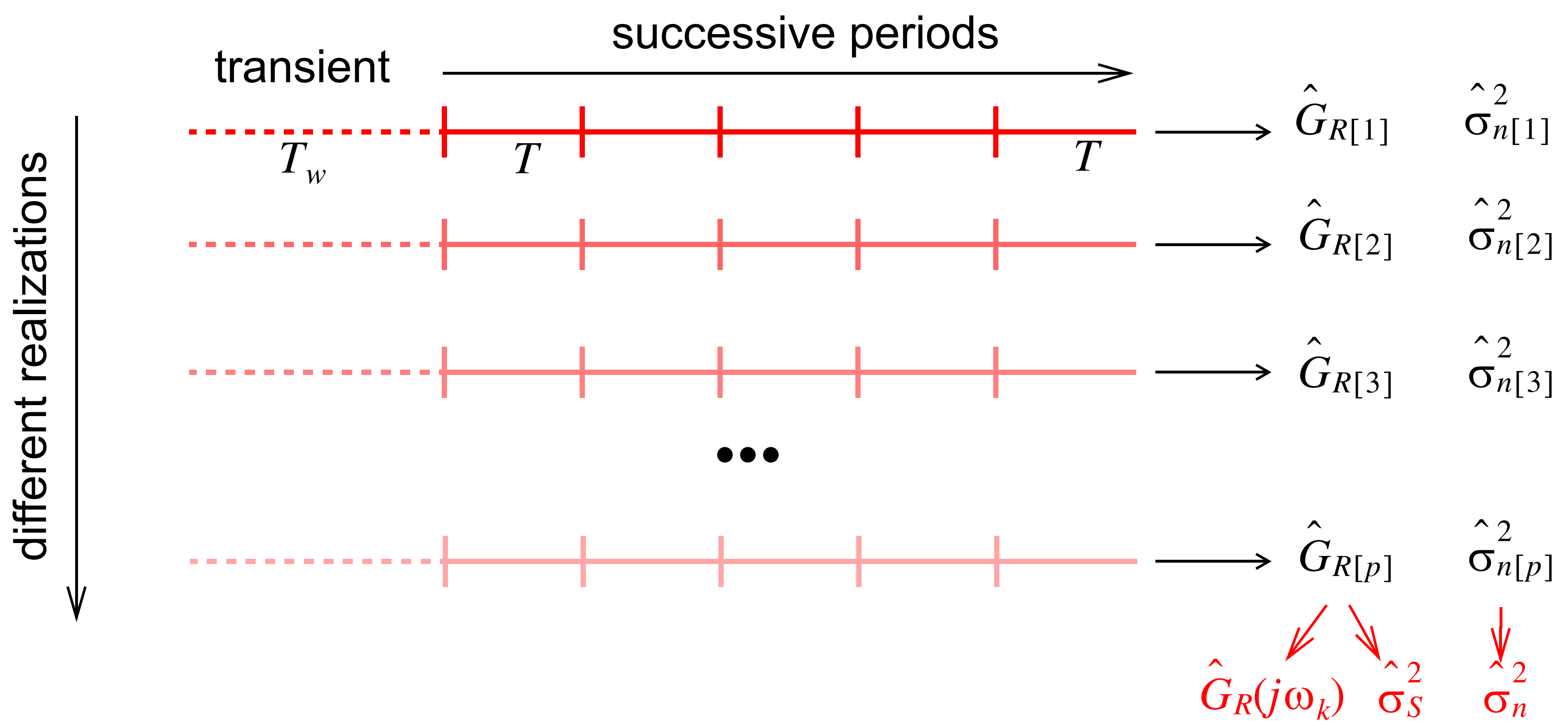


$$Y(k) = G_R(j\omega_k)U(k) + Y_S(k) + Y_N(k)$$

Method 1

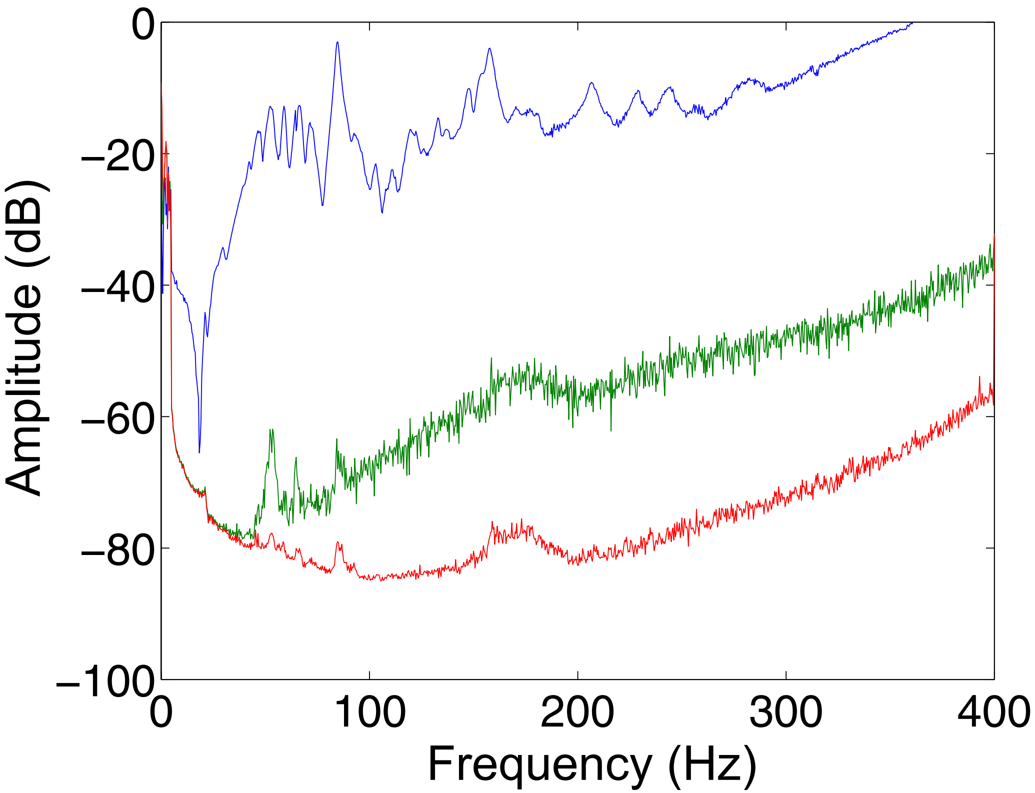
A robust approach



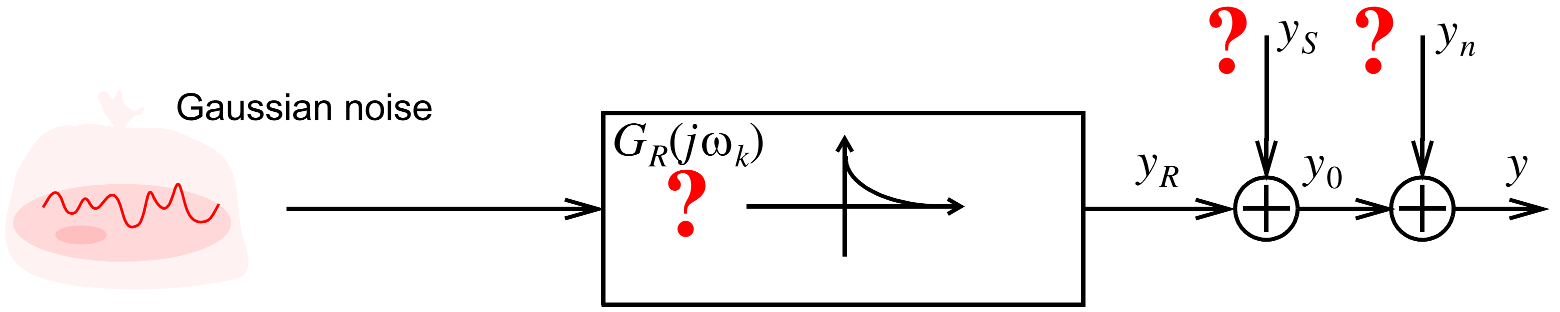


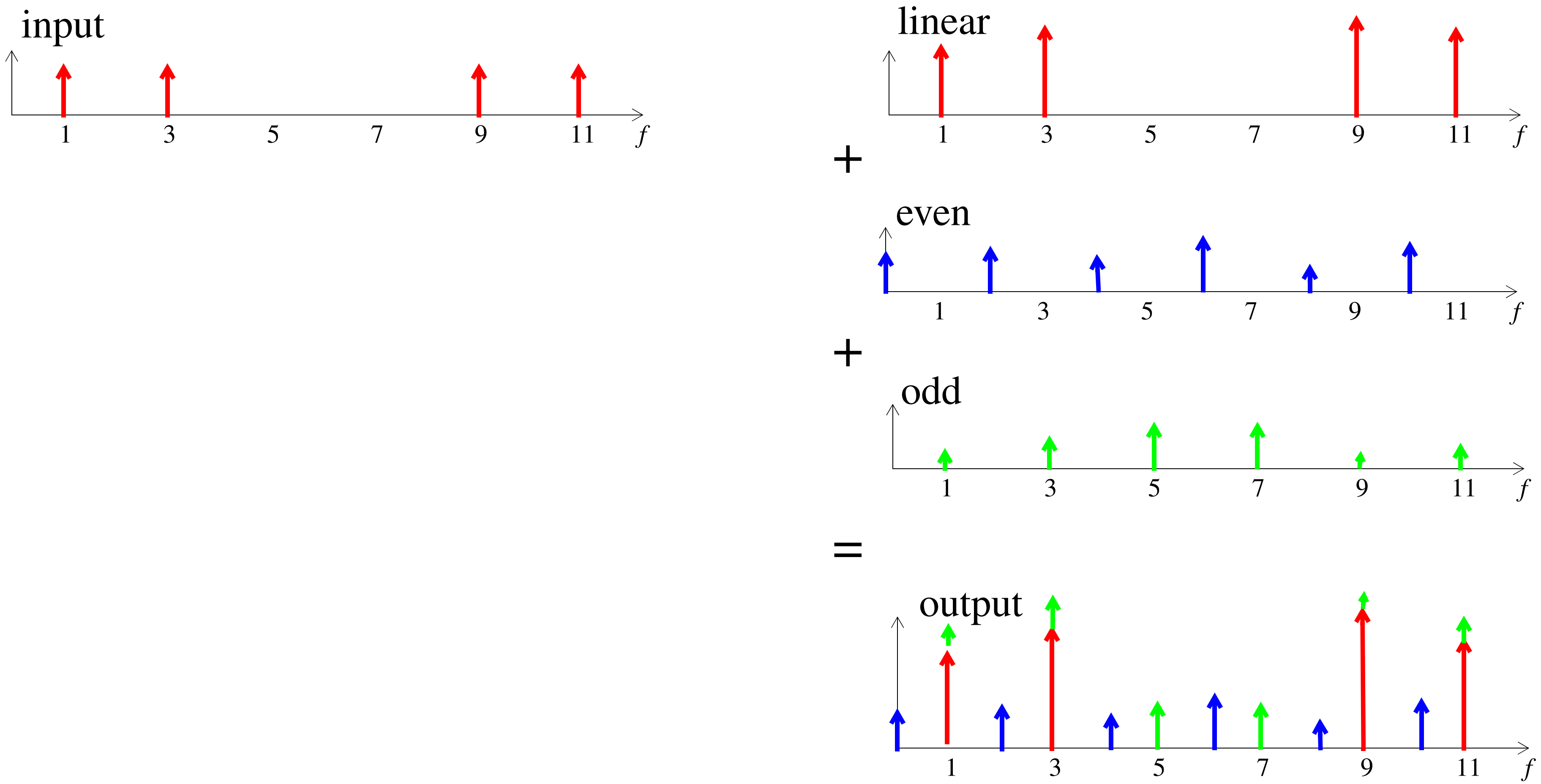
Example

(from Leuven Measurement Systems Int.)

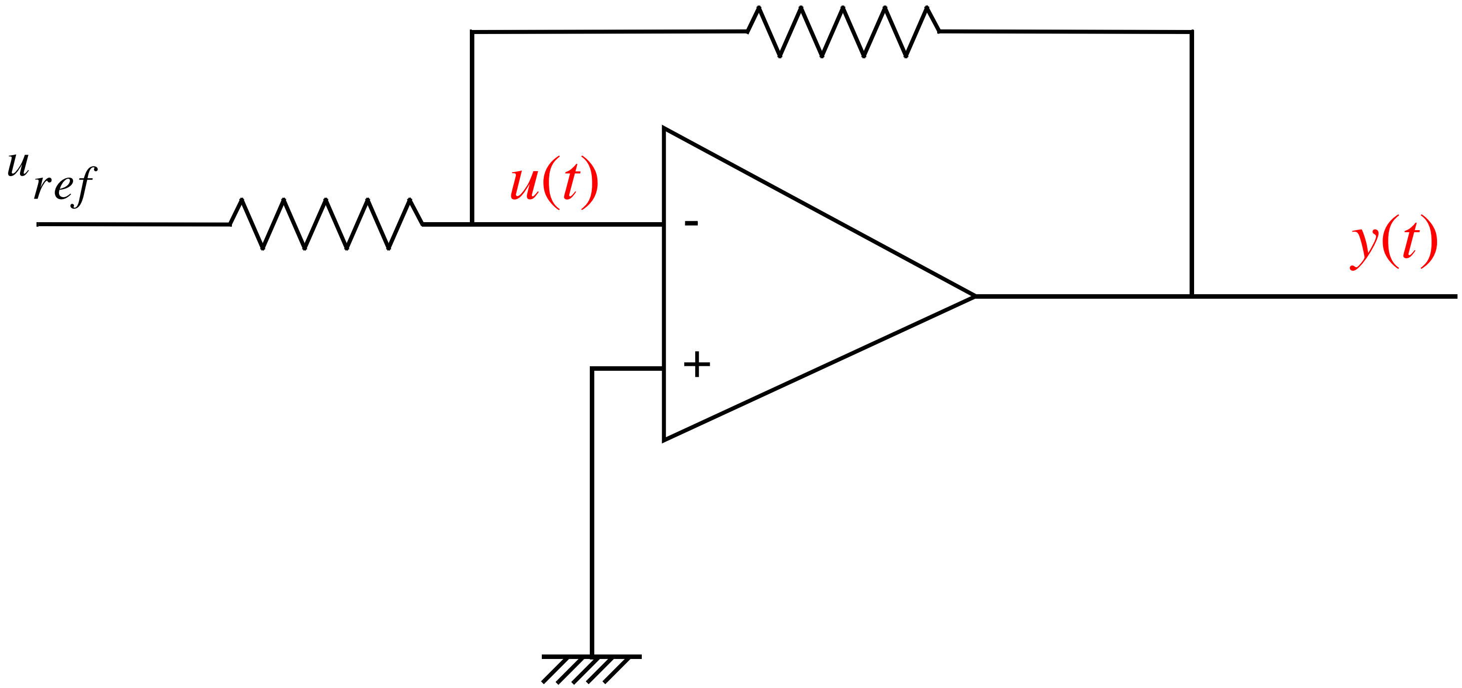


Method 2
Faster, more information, less robust

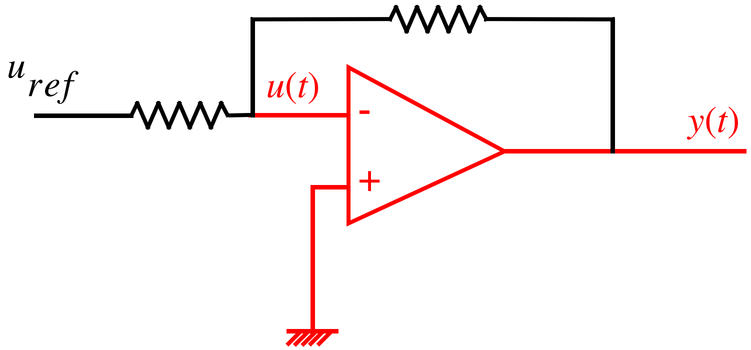




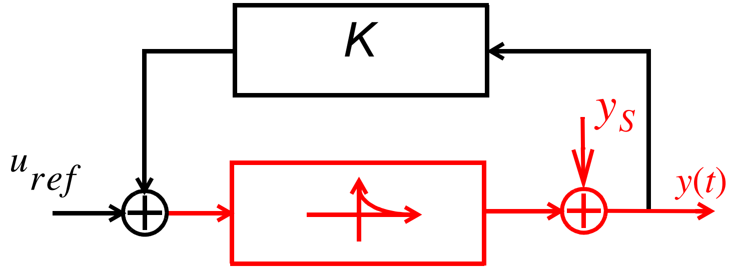
Example



Example cont'd: the FRF



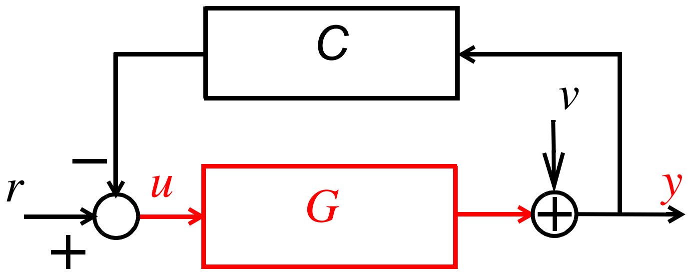
⇔



Interlude

FRF measurements in a feed back loop

$$U = SR - CSV$$
$$Y = GSR + SV$$
$$S = \frac{1}{1 + GC}$$



$$S_{YU} = G|S|^2 S_{RR} - C|S|^2 S_{VV}$$
$$S_{UU} = |S|^2 S_{RR} + |C|^2 |S|^2 S_{VV}$$

$$\hat{G} = \frac{S_{YU}}{S_{UU}} = \frac{GS_{RR} - CS_{VV}}{S_{RR} + |C|^2 S_{VV}}$$

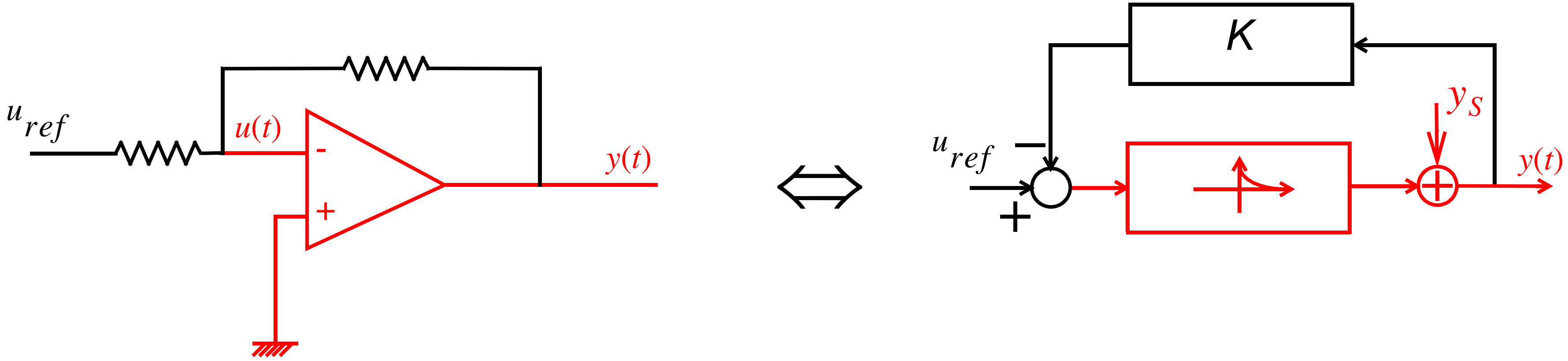
Reference signal dominates

$$\hat{G} \approx \frac{GS_{RR} - 0}{S_{RR} + 0} \approx G$$

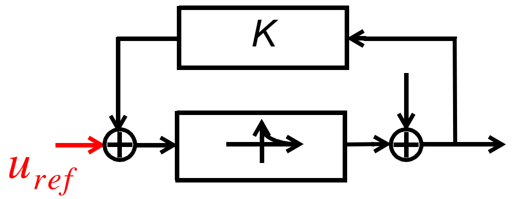
Noise signal dominates

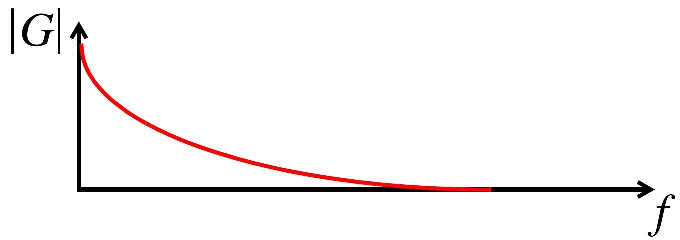
$$\hat{G} \approx \frac{0 - CS_{VV}}{0 + |C|^2 S_{VV}} = \frac{-1}{\bar{C}}$$

Example cont'd: the FRF

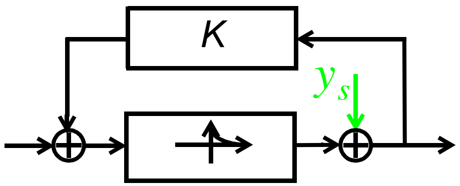


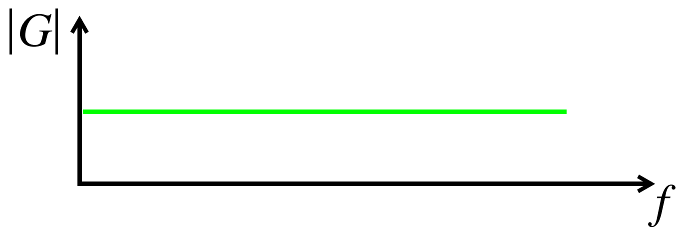
- u_{Ref} dominates: the feed forward is measured





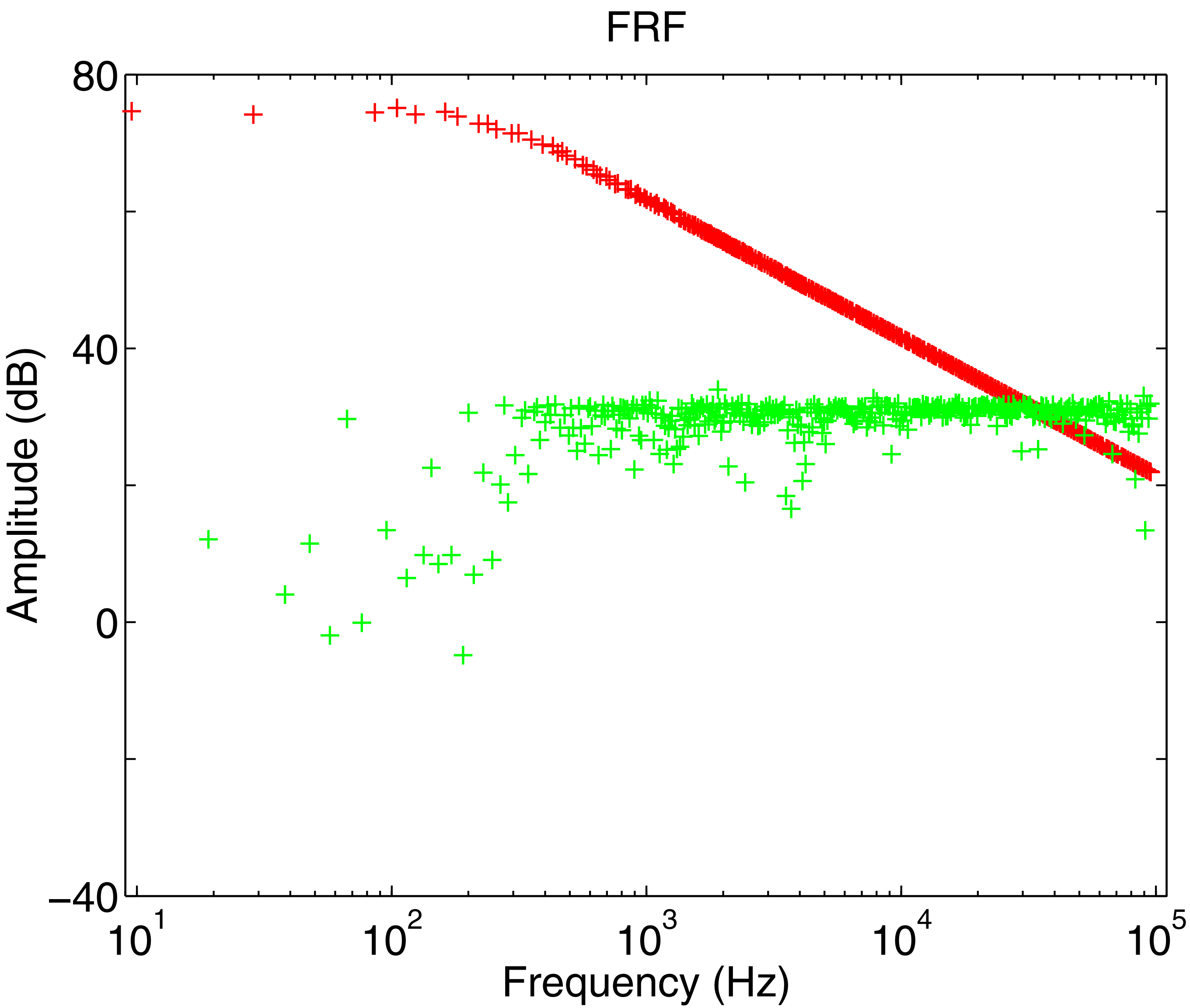
- y_S dominates: the inverse feed back K^{-1} is measured





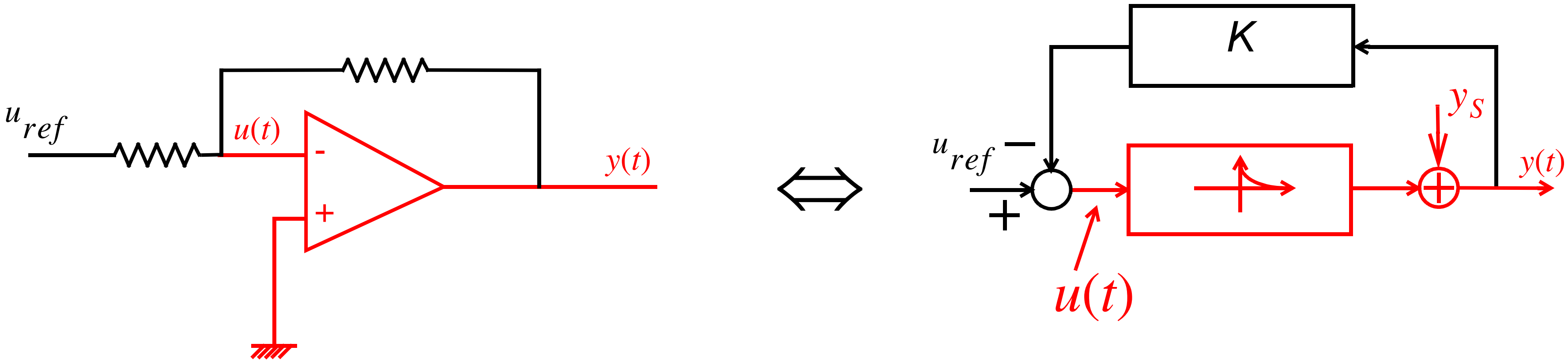
Example cont'd: the FRF

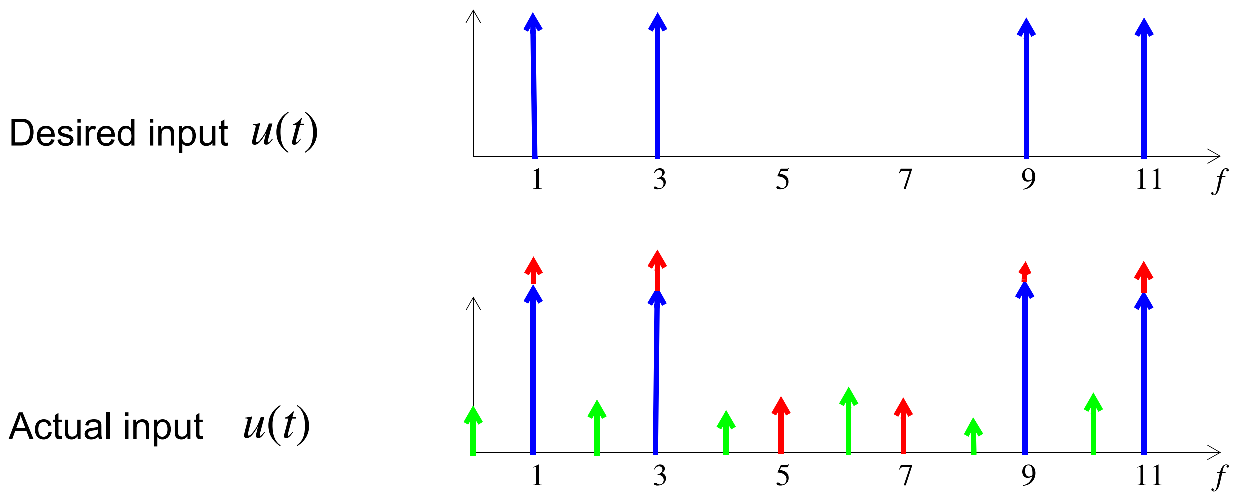
measurement results



Example cont'd: the nonlinear distortions

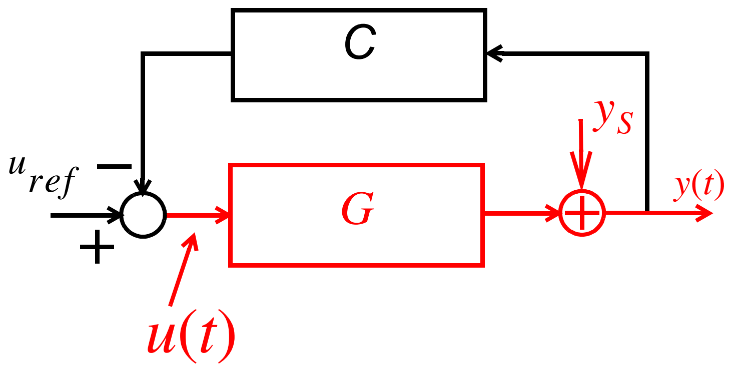
Problems

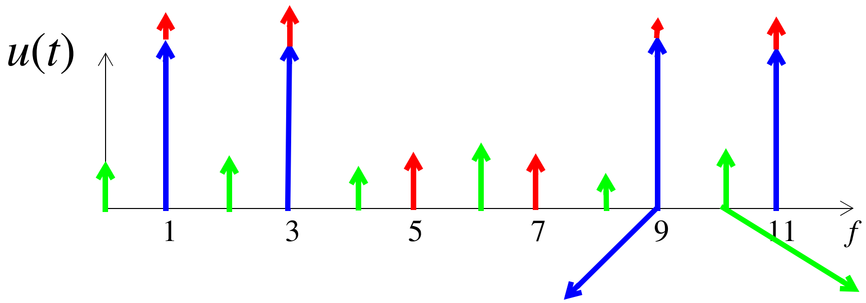




Example cont'd: the nonlinear distortions

Problems





U_{ref} dominant

$$\hat{G} \approx \frac{Y}{U}$$

Y_S dominant

$$U \approx \frac{-C}{1+CG} Y_S$$

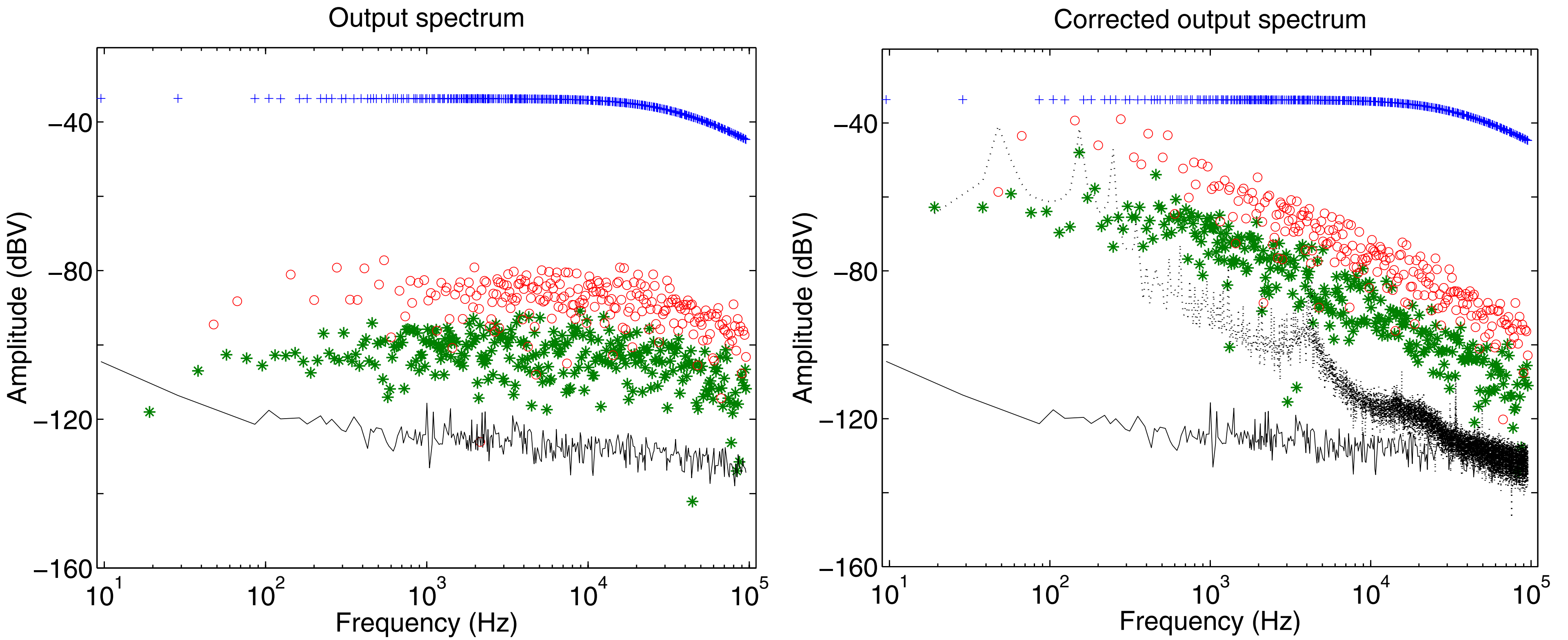
$$Y \approx \frac{1}{1+CG} Y_S$$

Correction

$$\begin{aligned} Y_{Sc} &= Y - \hat{G}U \\ &= \frac{1}{1+CG} Y_S - \frac{-CG}{1+CG} Y_S \\ &= Y_S \end{aligned}$$

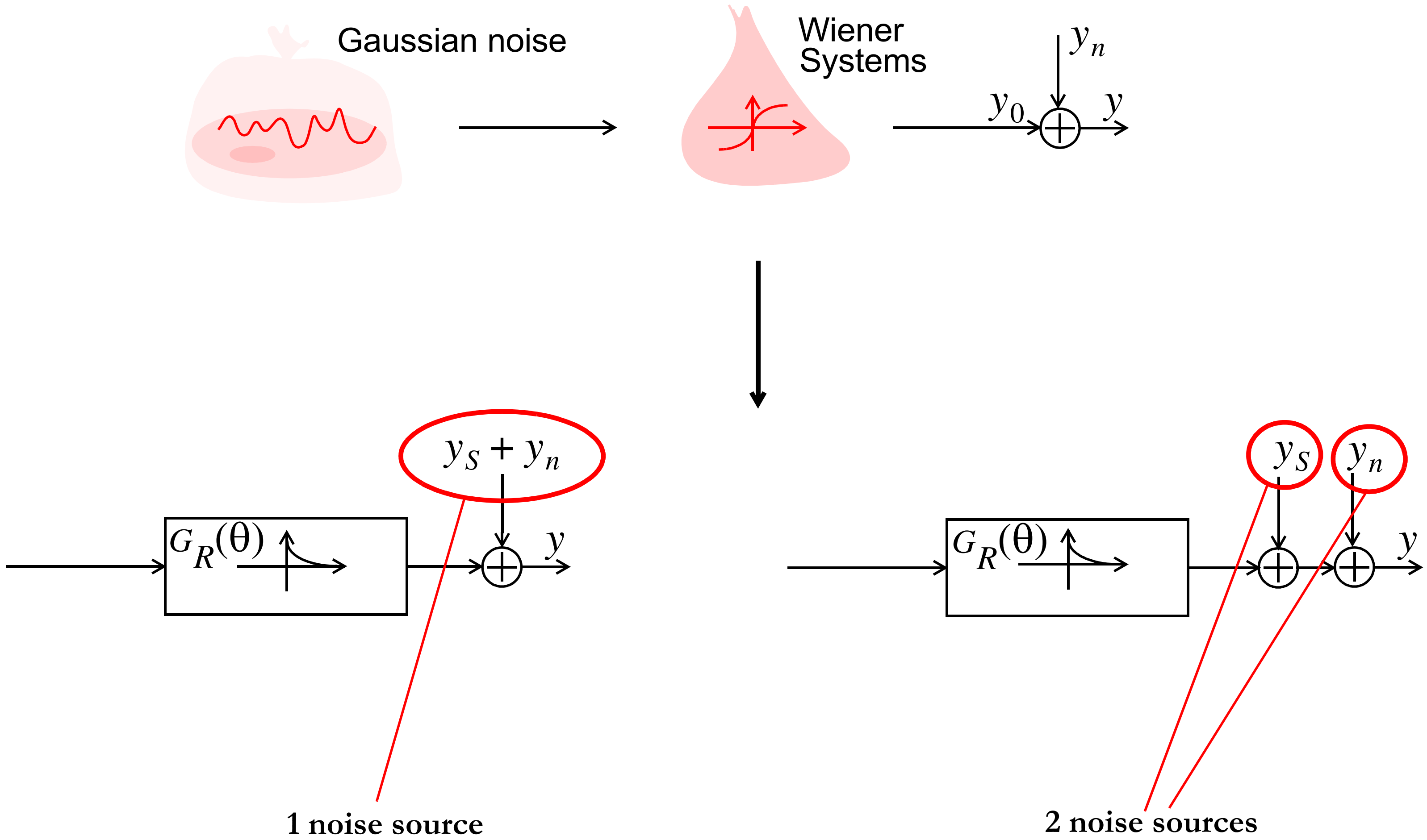
'Software opening' of the closed loop

Example cont'd: the nonlinear distortions at the output
Measurement results



Parametric linear modelling

Two possibilities



Parametric noise model Classical prediction error frame work	Non-parametric noise model
	Preprocessing - non-parametric noise model
Estimates - parametric plant model - parametric noise model (nonlinear and disturbing noise)	Estimates - parametric plant model
Properties - consistent - efficient - normal	Properties - consistent - efficient - normal
Validation - nonlinearity is NOT detected	Validation - nonlinearity is detected - alternative validation scheme
Happy but ‘unconscious’ user	**Happy but ‘conscious’ user**

Outline

Introduction

Approximation of nonlinear systems: important aspects

A nonlinear framework

Coherent and non coherent output contributions

A new paradigm

Detection, qualification, quantification of nonlinear distortions

Conclusions

Identification of linear systems with nonlinear distortions

Conclusions

Class of excitations

Class of systems

Approximations

?

Recycling linear identification

Nonlinear identification

Conclusions

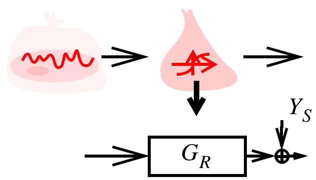
Class of excitations

Class of systems

Approximations

Recycling linear identification

Nonlinear identification



Next lesson

Frequency Response Function Measurements (February 8)

Impact of Nonlinear Distortions on the Linear Framework (February 15)

System Identification (February 22)

Identification of Linear Systems (March 1)

Identification of Nonlinear Systems (March 8)