# Static Nonlinearity Testing of Digital-to-Analog Converters

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***Abstract*—The paper presents a diagnostic tool for analyzing the bit intermodulations in digital-to-analog converters (DACs). Bit intermodulations cause linearity errors which degrade the performance of the converter. A better understanding of these errors can lead to designing and building more accurate converters. Therefore, a new static nonlinear model is proposed to incorporate intermodulation errors. A linear transformation of the Walsh transform of the integrated nonlinearity diagram (INL) is shown to be sufficient to extract the bit intermodulation terms and their noise sensitivity. Practical applicability of the proposed method is shown by measurements performed on a custom-designed test circuit.**

***Index Terms*—Bit intermodulation, digital-to-analog converter (DAC), testing data converters, Walsh transformation.**

## I. Introduction

THERE is a growing demand for highly linear digital-to-analog converters (DACs), as the significance of digitally controlled instruments and devices and complex modulations used for telecommunication is rapidly increasing. The linearity of DACs is highly dependent on—and limited by—the IC manufacturing processes.

However, the overall study of the converter's linearity can be done without going into technology-level analysis. The paper will determine a nonlinear model for a multiplying DAC.

The only needed *a priori* information is the *integrated nonlinearity diagram (INL)* of the data converter. The INL is defined as the difference of the real and ideal output characteristics of the converter.

## II. Linearity Errors of DACs

A perfectly linear DAC can be described by (1), which has zero *integral* and *differential nonlinearity*. Note that the global offset and gain errors do not have effects on the linearity properties of the converter

$$V_{\mathrm{out}} = V_{\mathrm{ref}} \cdot \sum_{i=1}^{N} \frac{b_i}{2^i}. \tag{1}$$

The device has two inputs: the analog input voltage $V_{\mathrm{ref}}$, and the digital input code $b_1 \cdots b_N$. The linearity errors may have multiple causes, and these errors are topology and sometimes process dependent. The most common reason for having nonlinear distortions of a converter is that the binary weights of the individual bits are not perfect. For example, if the converter uses the well-known R-2R ladder to generate binary weighted currents, resistor mismatches distort the binary divisions. Although the imperfection will cause linearity error, the *superposition principle* will still hold: the contribution of a bit to the output signal is independent from the states of the other individual bits, and therefore the superposition of the erroneous binary weighted currents—in reference to the input code—will give the actual output current [1]. In that case, the linearity of the device can be fully described by the bit error terms $\varepsilon_i$ for any digital input combination

$$V_{\mathrm{out}} = V_{\mathrm{ref}} \cdot \sum_{i=1}^{N} \left[ \frac{b_i}{2^i} (1 + \varepsilon_i) \right]. \tag{2}$$

Unfortunately, there are some errors which can not be described with the above formula. For example, the interactions between switches can produce nonlinearities for which the superposition principle no longer holds. These errors are called *superposition errors* in the literature [1], because the superposition of the corresponding erroneous bit currents will not give the actual output. In that case, a thorough test is needed to determine the worst case errors.

Extending the model of the converter (2) to cover the static superposition nonlinearities leads to a more accurate model of the circuit

$$\begin{aligned} V_{\mathrm{out}} = V_{\mathrm{ref}} \cdot \Bigg( &\sum_{i=1}^{N} \frac{b_i}{2^i} + \sum_{i=1}^{N} b_i \varepsilon_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} b_i b_j \varepsilon_{ij} \\ &+ \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sum_{k=j+1}^{N} b_i b_j b_k \varepsilon_{ijk} \\ &+ \cdots + b_1 \cdots b_N \varepsilon_{1 \cdots N} \Bigg). \end{aligned} \tag{3}$$

This model incorporates the effect of various switch interactions, and any other source of nonlinearity. The $\varepsilon$ terms describe

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the different interaction mechanism that span the superposition error.

In practice, the superposition errors are so small, that the linear error model (2) is often valid. However, as the resolution or the required linearity increases, the effect of the superposition errors can no longer be neglected.

## III. Analyzing the Linearity Errors

### A. Walsh Functions

The time domain analysis of the nonlinearity is useful to determine the specifications of the converter, but not for evaluating the error sources. This is especially true for superposition errors, which are easy to detect but whose source is almost impossible to trace. However, if the analysis is made in some another space, where the cumulative errors can be spread into orthogonal error components, it becomes possible to identify the error sources. The Walsh transform exactly performs such a decomposition [2]. The Walsh functions are defined as

$$W(n, t) = \prod_{r=0}^{N-1} (-1)^{n_{N-1-r}(t_r + t_{r+1})} \qquad (4)$$

where $n_k$ and $t_k$ are the $k$th-bit of the $N$-bit binary representation of numbers $n$ and $t$, and the least significant bits (LSBs) are denoted by $n_0$ and $t_0$. Variable $n$ is the sequence number of the Walsh function, and $t$ denotes the time. Due to the definition of the functions (4), the value of the functions can be $+1$ or $-1$. The finite discrete Walsh transform pair is defined similarly to the discrete Fourier transformation (DFT). Assuming that there are $2^N = m$ sampling points, the Walsh transform is

$$X_n = \frac{1}{m} \cdot \sum_{t=0}^{m-1} x(t) W(n, t) \qquad n = 0, 1, \ldots, m-1 \quad (5)$$

and the inverse transform is

$$x(t) = \sum_{n=0}^{m-1} X_n W(n, t) \qquad t = 0, 1, \ldots, m-1. \quad (6)$$

It is also possible to evaluate the transformation with a simple matrix multiplication. The transformation matrix contains the binary represented Walsh functions, and it is symmetrical. A further property is that the matrices of the transform pairs are identical except a scaling factor. For $m = 8$, the Walsh kernel functions are plotted in Fig. 1.

### B. Bit Errors

A common linearity error of converters is that the bit weightings are not ideal (2). If this nonideality is assumed to be dominant, then there is *no* superposition error. The number of error sources then equals the number of bits, and therefore from $N$ appropriate measurements the real transfer function can be derived. In practice, the superposition errors are so small, that this assumption is valid. However, as the resolution or the required linearity increases, the effect of the superposition errors can no longer be neglected.

First, bit errors will be studied. If a bit is switched on, then an erroneous current value appears at the output. This means that if there is only one bit error, the INL looks like a square wave whose amplitude equals the amplitude of the error and with a frequency which depends on the position of the erroneous bit (Fig. 2). It is a consequence of the definition of the INL, which is the difference between the real and ideal output characteristic.

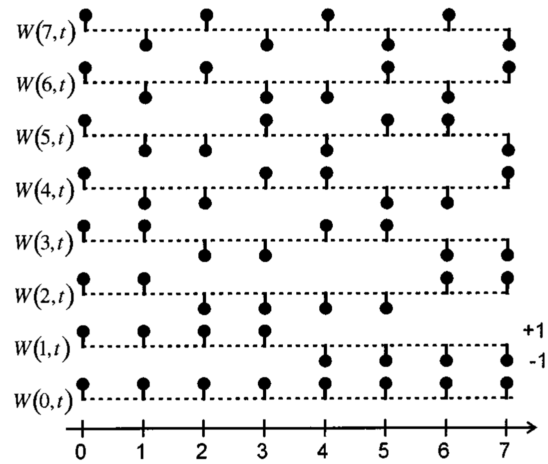


Fig. 1. Walsh functions for $m = 8$.

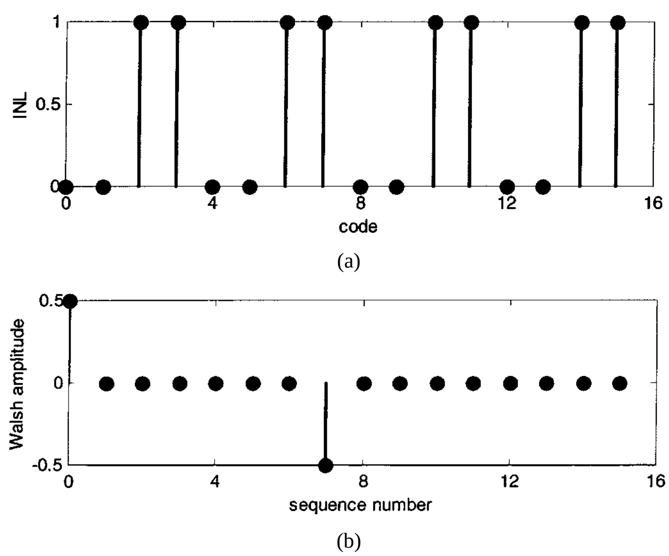


Fig. 2. Four-bit converter's INL diagram and its Walsh transform with one bit error ($\varepsilon_3 \neq 0$).

Fortunately, a square wave can easily be described by Walsh functions, because it is one of the orthogonal basis functions of the transformation (Fig. 1). The above example can be described by a $W(7, t)$ Walsh function and a $W(0, t)$ function representing the offset. It can be calculated, that if the $i$th ($i = [1, \ldots, N]$) bit has an error then it will generate a Walsh component at sequence $2^i - 1$ and at sequence zero. The amplitude of the Walsh sequence will be one-half of the error. This is due to the definition of the Walsh functions [they are defined and orthonormal over the set $(+1, -1)$ and not over $(+1, 0)$].

Hence, the bit errors are easily detectable in the Walsh transform of the INL: just the $2^i - 1$ sequences must be calculated for each $i$. The effect of the noise is decreased with this method, because in the Walsh space there are only $N$ nonzero sequences from $2^N$ components. However, many more samples are measured than it would be minimally required. The benefit of analyzing the INL in the Walsh space becomes evident if higher order interactions are also present.

### *C. Higher Order Intermodulations*

In the remaining sections, the number of interacting bits will be referred as the *order* of intermodulation.

Due to the higher order intermodulations, there are superposition errors present,—and perhaps bit errors, as well—which must be characterized. If two bits are interacting, then an error function will result which is the product of the two corresponding bit error functions, which can also be described by the *product* of two Walsh functions. From the definition of the Walsh functions it can be easily seen that the product is equal to

$$W(n,\, t)W(h,\, t) = W([n \oplus h]),\, t) \tag{7}$$

where the $\oplus$ operator is the modulo-2 summation. This means that the multiplication of two Walsh functions produces another Walsh function. On the other hand, due to the modulo-2 summation, the multiplication will not generate a sequence outside the $(0,\, 2^{N-1})$ space. However, again caused by the definition of the Walsh series, the real result will be slightly different. The error function of a bit error is described by two Walsh functions

$$\varepsilon_{i_n} = \left(-\tfrac{1}{2}\right)[W(n,\, t) - W(0,\, t)] \tag{8}$$

where $n = 2^{i_n} - 1$ and $i_n \in [1,\, \ldots,\, N]$ denotes the index of the erroneous bit. Therefore, the product of two error functions in the time domain is

$$\begin{aligned}
&\varepsilon_{i_n}(t)\varepsilon_{i_h}(t)\\
&\quad = \frac{1}{2^2}[W(n,\, t)W(h,\, t) - W(n,\, t)W(0,\, t)\\
&\qquad\quad - W(h,\, t)W(0,\, t) + W(0,\, t)W(0,\, t)]\\
&\quad = \frac{1}{2^2}[W([n \oplus h],\, t) - W(n,\, t) - W(h,\, t) + W(0,\, t)]\\
&\quad = \varepsilon_{i_n i_h}(t).
\end{aligned} \tag{9}$$

From this simple calculation, it is clear that the intermodulation will produce components also at the sequences where the bit errors $(W(n,\, t),\, W(h,\, t))$ should appear.

Although it is possible to have interactions between any bits of the converter, the significant correlations will appear between physically neighboring bits, which have in most cases neighboring weightings. This important case will be analyzed in detail. Suppose that there is intermodulation between two neighboring bits of the $N$-bit converter, $i$ and $i+1$. The corresponding Walsh components of the bits are at sequence $2^i - 1,\, 2^{i+1} - 1$. The modulo-2 summation of these two terms will give the sequence number of their product.

This results in a Walsh function at sequence $2^{N-i-1}$. The intermodulation of the two bits also causes Walsh components at sequences $2^i - 1,\, 2^{i+1} - 1$ [see (9)]. Therefore, if there are bit errors and bit intermodulations at the same time, then the weights of the Walsh components of the bit errors are affected by bit errors and intermodulations, as well (see Fig. 3).

Just as for 2-bit intermodulation, the higher order intermodulations can also be determined with the Walsh functions. For calculating the error function of a higher order intermodulation the product of more Walsh functions is needed. Each function in the product corresponds to a bit taking part in the interaction.

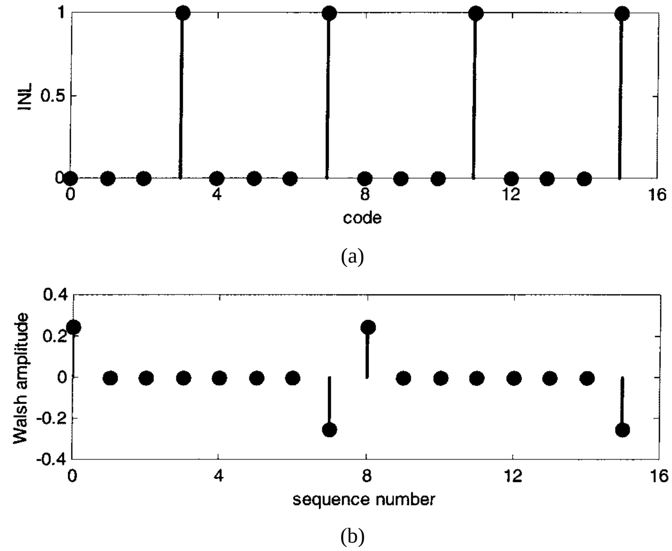


Fig. 3. Bit-converter with two intermodulating bits $(b_3,\, b_4)$ and without bit errors.

The product of more Walsh functions is similar to the former case

$$W(n_1,\, t) \cdot \cdots \cdot W(n_r,\, t) = W([n_1 \oplus \cdots \oplus n_r],\, t). \tag{10}$$

The result of multiple product terms is a Walsh function with sequence number calculated as the modulo-2 sum of the product sequences. Again, an intermodulation generates more than one nonzero component, and components will also appear at *all* of the lower order intermodulations

$$\begin{aligned}
\varepsilon_{i_{n_1}} \cdot \cdots \cdot \varepsilon_{i_{n_r}}(t) &= \prod_{j=1}^{r} \left(-\tfrac{1}{2}\right)[W(n_j,\, t) - W(0,\, t)]\\
&= \left(-\tfrac{1}{2}\right)^r \prod_{j=1}^{r}[W(n_j,\, t) - W(0,\, t)].
\end{aligned} \tag{11}$$

Hereafter, the expression *mainterm* will refer to the sequence calculated as the modulo-2 sum of all the interacting bits $(n_1 \oplus \cdots \oplus n_h)$. This sequence is only affected by higher order intermodulations, and not by lower or equal ones. This *key fact* enables the separation of the error sources. Unfortunately, different intermodulations can be present at a single sequence (except the highest order mainterm), but they can be separated by solving a set of linear equations. However, this is at the cost of an increased noise sensitivity. From (11), it is also clear that the amplitude of an $r$-order intermodulation will be decreased by a factor of $1/2^r$.

### *D. Fourier or Walsh Transformation Based Diagnostics?*

It would also be possible to use the well-known DFT for detecting the different error sources in the nonlinearity diagrams. However, there are some great benefits of using the Walsh functions instead of the Fourier series, as it will be discussed now.

- The significant bit error functions are some of the kernel Walsh functions. This is very important, because they can be easily detected by Walsh transformation. However, the Fourier transformation generates already a “complex” spectra in case of bit errors only, where the separation of the different errors is hard.

- Due to the modulo-2 summation, the product of Walsh functions will produce a component in the same Walsh space, so all kinds of transform products can be described in the $(0,\ 2^N-1)$ Walsh space. In case of Fourier analysis, the transform product produces two components, which makes the identification more difficult.

## IV. Transformation of the INL into Error Terms

### A. Transformation Matrix

Although the Walsh transformation separates the first-order errors ideally, for higher order intermodulations, an additional linear transformation is needed due to the spreading of higher order intermodulation terms through the Walsh space. The identification of the intercorrelation terms requires two steps. First, the Walsh spectra of the INL are calculated

$$\underline{w} = \mathbf{W} \cdot \underline{\mathrm{inl}} \tag{12}$$

where $\mathbf{W}$ is the Walsh transformation matrix. The spectra are then further transformed with a linear transformation $\mathbf{T}$ in the intermodulation terms

$$\underline{\varepsilon} = \mathbf{T} \cdot \underline{w} \tag{13}$$

where $\underline{\varepsilon}$ contains the intermodulation terms. There are only $2^N - 1$ intermodulations in the $\underline{\varepsilon}$ vector, because the zeroth-order intermodulation has no importance. For a better understanding a simple example will also be given during the analysis. The following vector contains the correlation terms for a 3-bit converter (the three bits are denoted as 1, 2, and 3)

$$\underline{\varepsilon}^{\mathbf{T}} = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \varepsilon_{12}\ \varepsilon_{13}\ \varepsilon_{23}\ \varepsilon_{123}]. \tag{14}$$

The calculation of the $\mathbf{T}$ matrix is needed for characterizing the linear transformation. First, new vectors will be defined, which will contain the information we know about the intermodulations. As it was stated before, the mainterm—which is the modulo-2 summation of the sequence numbers of the bits taking part in the intermodulation—is not affected by higher order correlations. To extract the mainterm of one specific intermodulation from the Walsh space, let us define new vectors $(\underline{a}_k)$ for each intermodulation with the following equation:

$$w_g = \underline{a}_k^{\mathbf{T}} \underline{w} \tag{15}$$

where $g$ is the mainterm sequence of intermodulation $k$. Therefore, $\underline{a}_k$ contains zeros except the sequence $g$. The vectors will look for a 3-bit converter as

$$[\underline{a}_1 \cdots \underline{a}_7] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{16}$$

Let us start with the highest order intermodulation. It creates a sequence according to $\underline{a}_{m-1}$, which is specific for this intermodulation. For simplicity $m$ again denotes $m = 2^N$. To restore the magnitude of the error, the value at this sequence must be multiplied with $2^N$ (generally with $2^r$ where $r$ is the order of intermodulation). So the amplitude of the intermodulation can be calculated as

$$\varepsilon(m-1) = 2^N \underline{a}_{m-1}^{\mathbf{T}} \underline{w} \tag{17}$$

where $\varepsilon(i)$ denotes the $i$th element of vector $\underline{\varepsilon}$ ($\varepsilon_i$ is reserved for the intermodulation error terms). The next intermodulation is calculated similarly to (17), just the higher order intermodulation must be neglected

$$\varepsilon(m-2) = 2^{N-1} \cdot \left[\underline{a}_{m-2}^{\mathbf{T}} - \underline{a}_{m-1}^{\mathbf{T}}\right] \underline{w}. \tag{18}$$

The next term will be

$$\varepsilon(m-3) = 2^{N-1} \left[\underline{a}_{m-3}^{\mathbf{T}} - \underline{a}_{m-1}^{\mathbf{T}}\right] \underline{w}. \tag{19}$$

The $(m-i)$th term will be equal to

$$\varepsilon(m-i) = 2^r \left[\underline{a}_{m-3}^{\mathbf{T}} - \sum_{j \in H_r} \underline{a}_j^{\mathbf{T}}\right] \underline{w}. \tag{20}$$

The set $H_r$ contains the indexes of higher order intermodulation ($j$ is not equal to $m-i+1$ for any case). The above equations can be interpreted in a final compact form

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon(1) \\ \cdots \\ \varepsilon(m-2) \\ \varepsilon(m-1) \end{bmatrix} = \begin{bmatrix} 2\left[\underline{a}_1^{\mathbf{T}} - \sum_j \underline{a}_j^{\mathbf{T}}\right] \\ \cdots \\ 2^{N-1}\left[\underline{a}_{m-2}^{\mathbf{T}} - \underline{a}_{m-1}^{\mathbf{T}}\right] \\ 2^N \underline{a}_{m-1}^{\mathbf{T}} \end{bmatrix} \underline{w} = \mathbf{T}\underline{w}. \tag{21}$$

The result is not really suitable for further synthesis, but it is quite easy to implement. For a 3-bit converter the $\mathbf{T}$ transformation matrix will be equal to

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & -2 & -2 & -2 & -2 \\ 0 & 0 & -2 & -2 & -2 & -2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 \end{bmatrix}. \tag{22}$$

### B. Noise Analysis

The noise sensitivity is characterized by the condition number of $\mathbf{T}$. Table I contains the condition numbers for some cases.

TABLE I
CONDITION NUMBERS OF $\mathbf{T}$ FOR DIFFERENT RESOLUTIONS

| $N$ | cond($\mathbf{T}$) |
|---|---|
| 4 | 25.9869 |
| 6 | 197.1629 |
| 8 | 1456.8000 |

However, the condition numbers are a worst-case estimation of the noise sensitivity. Using the standard deviation of the measurement results, a narrower bound on the noise influence can be found. Assuming additive noise disturbing the measurements, the covariance matrix of the noise $(\mathbf{C}_{\rm inl})$ is transformed during the transformations

$$\begin{aligned}\mathbf{C}_{\varepsilon} &= \mathrm{E}\left\{\underline{\varepsilon}\,\underline{\varepsilon}^{\mathbf{T}}\right\} = \mathrm{E}\left\{\mathbf{T}\underline{w}\,\underline{w}^{\mathbf{T}}\mathbf{T}^{\mathbf{T}}\right\}\\ &= \mathrm{E}\left\{\mathbf{T}\mathbf{W}\underline{\rm inl}\,\underline{\rm inl}^{\mathbf{T}}\mathbf{W}\mathbf{T}^{\mathbf{T}}\right\}\\ &= \mathbf{T}\mathbf{W}\cdot\mathrm{E}\left\{\underline{\rm inl}\,\underline{\rm inl}^{\mathbf{T}}\right\}\cdot\mathbf{W}^{\mathbf{T}}\mathbf{T}^{\mathbf{T}}\\ &= \mathbf{T}\mathbf{W}\mathbf{C}_{\rm inl}\mathbf{W}^{\mathbf{T}}\mathbf{T}^{\mathbf{T}}. \end{aligned} \tag{23}$$

If the noise on the INL is uncorrelated, then its covariance matrix will be diagonal. The Walshtrans formation is an orthogonal transformation, and therefore $\mathbf{W}\mathbf{W}^{\mathbf{T}}$ is also diagonal. In that case, (23) simplifies further

$$\mathbf{C}_{\varepsilon} = \mathbf{C}_{\rm inl}\mathbf{T}\mathbf{W}\mathbf{W}^{\mathbf{T}}\mathbf{T}^{\mathbf{T}} = \mathbf{C}_{\rm inl}\frac{1}{m}\mathbf{E}_{m}\mathbf{T}\mathbf{T}^{\mathbf{T}}. \tag{24}$$

The Walsh transformation reduces the variance by a $1/m$ factor. Although it looks impressive at the first look, it should be mentioned that the signal-to-noise ratio will remain the same, because the signal levels are also reduced by the transformation. The covariance on the intermodulation terms will be significantly changed by the $\mathbf{T}$ transformation. The $\mathbf{T}$ matrix is not orthogonal, and therefore $\mathbf{T}\mathbf{T}^{\mathbf{T}}$ is not diagonal. Owing to this fact, the covariance matrix $\mathbf{C}_{\varepsilon}$, even if $\mathbf{C}_{\rm inl}$ is diagonal, will not be diagonal. On the other hand, it means that even for uncorrelated input noise, the output noise will be correlated. To further analyze the noise on the output, the $\mathbf{T}\mathbf{T}^{\mathbf{T}}$ matrix must be studied. It is quite hard to express it in an analyzable form. But it can be stated, that the largest element of the $\mathbf{T}\mathbf{T}^{\mathbf{T}}$ matrix will be due to the highest order intermodulation, and this value is $2^{2N}$.

The noise analysis is important to have an idea about how many different realizations are required to achieve a user-defined uncertainty on the measurements. As it is known, the averaging of $k$ different realizations reduces the variance by a factor of $1/k$. The Walsh transformation generally does not change the signal-to-noise ratio (in practice it will reduce depending on the shape of the signal). However, the transformation described by $\mathbf{T}$ causes some problems. Because the $\mathbf{T}\mathbf{T}^{\mathbf{T}}$ matrix is not orthogonal, the transformed covariance matrix of the noise will not be diagonal. Therefore, the noise on the intermodulation terms will be correlated.

## V. Measurement Results

Although simulations were made to prove the theory, the final verifications should be made on real measurement data. Therefore, a test circuit was built, containing an 8-bit DAC and the support circuitry. The input data was generated by a digital I/O card, and the output was measured with the HP3458A high-precision multimeter.

The first measurements were simple INL diagram measurements: all codes were incrementally applied to the DAC. The reference signal was always a 10-V dc voltage, produced by a highly-stable, precision Zener reference. The INL was calculated as the difference between the real output and the best-fit line of the output characteristic. The INL measurements showed that the method is applicable. The next sets of measurements were made with different "random" excitations. The random means that although all combinations were excited during a period, the input signal's consecutive samples were chosen randomly. Three different realizations were measured, and each realization consisted of ten periods. The aim of these experiments was to distinguish between dynamic and static effects. If the results are independent on the input sequence, then the errors are related to static nonlinearities. If there is a change in the intermodulation terms between the different perturbations, then the errors are mainly caused by a dynamic effect. In that case, the method is practically unusable. In Fig. 4[1] the above discussed measurement results are plotted. The abscissa contains all the error terms, going from the bit-errors (index 1–8) to the higher order correlations. The interesting part is the first- and second-order intermodulations, which are magnified in Fig. 5. The three different realizations produce three *slightly* different errors. The bit errors are increasing from the MSB to the LSB, and this is in agreement with our expectation. To guarantee the specifications, the more significant bits should be more precise, and the lower ones less precise. This can be seen in Fig. 5. On the other hand, there are significant second-order intermodulations, especially at $\varepsilon_{78}$.

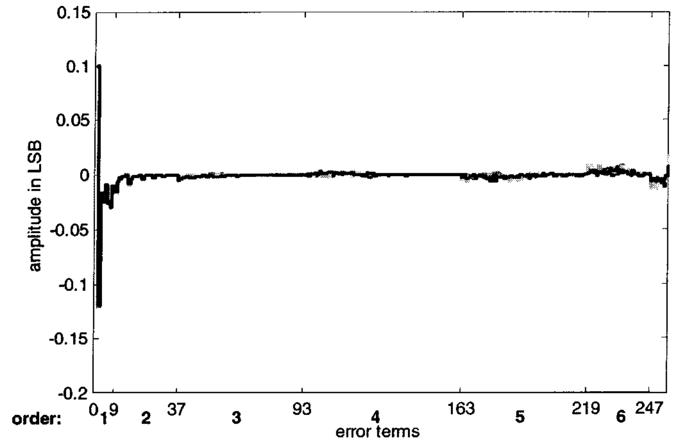


Fig. 4. All intermodulation error terms, "random" excitations.

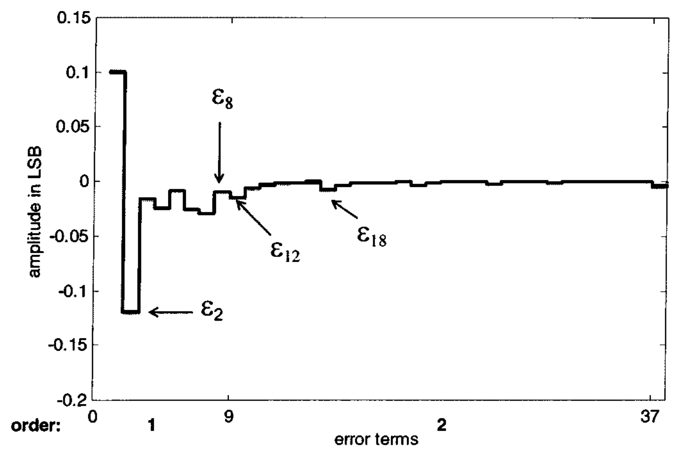


Fig. 5. First- and second-order intermodulation error terms, "random" excitations.

[1]These figures are plotted with the MatLab *stairs* function, therefore $f(x_i)$ value is plotted as a line from $x_i$ to $x_{i+1}$.

## VI. CONCLUSIONS

A new method is presented to characterize the static nonlinearities of DACs. It was shown that the INL contains all the required information, and with the Walsh transformation, it is possible to identify the bit errors, as well as higher order intermodulations. However, due to the definition of the Walsh functions, higher order intermodulations also produce components at lower order terms. Therefore, another linear transformation is needed to extract the required information.

The measurement results verified the theory, and opened new research directions. Measurements with different amplitudes of dc input voltages and ac input voltages at varying frequencies are expected to yield further insight in the behavior of DAC nonlinearity.

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