

REPRESENTATION OF SISO FUZZY LOGIC CONTROL SYSTEMS WITHIN NLQ THEORY

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ABSTRACT

This paper gives an application of NL_q theory on Single Input Single Output (SISO) Fuzzy Logic controllers (FLC). We assume that a controlled plant can be accurately described (Certainty Equivalence Principle) using a linear or nonlinear model, e.g. a Fuzzy Logic System (FLS). An easy to use stability theorem is given, based on the properties of fuzzy controllers that use triangular and trapezoidal shaped Fuzzy Sets. Although the given criterium is rather conservative, sharper criteria are available within NL_q , but those are not addressed within the scope of this paper. The stability theorem is useable for both autonomous and non-autonomous systems that are used with an FLC, and an example is given to illustrate the theory.

I. INTRODUCTION

Since Zadeh [21] introduced Fuzzy Sets, many discussions have taken place whether Fuzzy Logic (FL) deserves a place in control theory. Three properties speak in favour of FL control. The first being its robustness against parameter uncertainty [3] [12], the second the fact that the FL controller output is normalized. While linear PID controllers assume that the controller output can be much larger than the true maximal input for a system, Fuzzy Logic Controllers (FLC) by nature restrict the controller value to a normalized interval. The third property in favor of FLC is the “linguistic” interpretation of the control scheme [9].

Lately much effort has been put to overcome the major drawback of FL systems: how to find an easy to use stability theorem? Although Fuzzy Systems (FS) are locally linear, the global transfer function is of a nonlinear nature. This paper discusses the stability of dynamical systems that can be described with either a linear model [11] [13] or a FL model [2] [17]. Although stability of FS has been addressed before, most papers focus on the local-linear properties of Takagi-Sugeno controllers [6] [18] [20]. In this paper we study the stability of Mamdani FLC, with a few assumptions on normalization of the input and output parameters as it was also done in [1], [8] and [18]. A sufficient proof will be given for the stability of Fuzzy Additive Systems [7].

NL_q theory has been introduced in [15] and [16] basically as a stability theory for multilayer recurrent neural networks with conditions for global asymptotic stability and input-output stability with finite L_2 -gain. It can be either used a theory for nonlinear H_∞ control of recurrent neural networks or as a method for imposing stability in dynamic backpropagation. Within this framework standard plant forms can be considered with process noise and measurement noise. Moreover links with μ -robust control theory have been established by considering parametric uncertainties upon NL_q systems. Many of the NL_q stability criteria can be expressed in terms of matrix inequalities. A real life application to a ball and beam system has been given in [19].

This paper is organized as follows. Section II of this paper introduces a Mamdani type of FS and describes this system in the scope of NL_q theory. A sufficient proof of stability for these systems is given in section III. To demonstrate the method section IV gives an example on a hybrid system with linear and FL parts.

II. FUZZY LOGIC SYSTEMS WITHIN THE NLQ FRAMEWORK

This section first discusses a paradigm for the description of a type of Mamdani Fuzzy Logic Systems (FLS). Basic assumptions are made about the Fuzzy Sets and the Knowledge Base of the FLS. The FLS is then written in a more general form as a concatenation of a linear system and a pure FS. After the introduction of the FLS, NL_q systems [14] are briefly introduced and the FLS is formulated as an NL_2 system.

A. Representation of a SISO Fuzzy System

Consider a SISO Fuzzy Additive System [7] $F: \mathbb{R} \rightarrow \mathbb{R}$ that stores r rules of the form

$$R_j: \text{If } x = A_j \text{ then } y = B_j. \quad (1)$$

The input fuzzy sets A_j and output fuzzy sets B_j are defined as in figures 1a and 1b. All “then” parts B_j are summed to give the output set $B(y)$

$$B(y) = \sum_{j=1}^r \mu_j(x) B_j(y). \quad (2)$$

The membership function $\mu_j(x)$ with $\mu_j: \mathbb{R} \rightarrow [0, 1]$ measures the degree to which the input x belongs to the fuzzy set A_j . The translation from x to $\mu_j(x)$ is the fuzzification part of the FLS.

Assumption 1 The system inputs and outputs are normalized, such that $x \in [-1, 1]$ and $y \in [-1, 1]$.

Assumption 2 The input sets are normalized such that the following equation holds

$$\sum_{i=1}^m \mu_i(x) = 1 \quad \forall x \in \mathbb{R} \quad (3)$$

with m the number of input sets. Typical fuzzy sets that comply with this assumption, are the combination of triangular and trapezoidal sets as shown in Fig. 1a. The first and last sets are chosen as half sets, so that they are open at the sides.

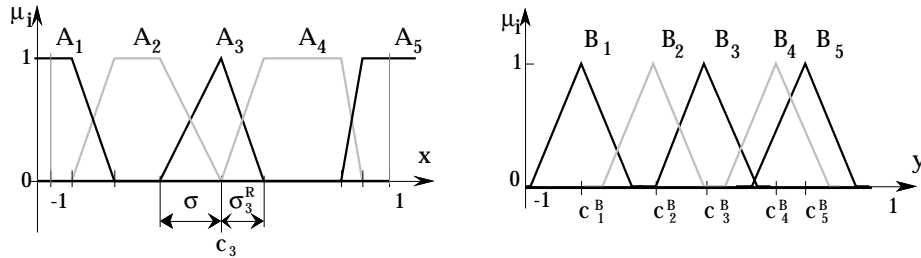


Fig. 1 a) Normalized fuzzy input sets

b) Output sets with a constant area

Assumption 3 All output sets B_j have the same area

$$b_j = \int_{\mathbb{R}} B_j(y) dy = b. \quad (4)$$

This assumption can easily be achieved by choosing the same base set for each output set B_j , as shown in Fig. 1b. The centers c_j^B of each output set are defined as

$$c_j^B = \frac{\int_{\mathbb{R}} y B_j(y) dy}{\int_{\mathbb{R}} B_j(y) dy} = \frac{1}{b} \int_{\mathbb{R}} y B_j(y) dy. \quad (5)$$

The generalized output sets B_j have an output function $B_j(y): \mathbb{R} \rightarrow [0, 1]$, such that the rules R_j form an arbitrary linear or nonlinear mapping $R_j: \mathbb{R} \rightarrow \mathbb{R}$.

Assumption 4 The rule base is full and non-redundant within the input domain.

For a SISO system, this implies that each input set is mapped to exactly one rule and the number of rules equal the number of input sets ($r = m$). The assumption that the rule-base is non-redundant implies that no contradictory rules are used

for the same input set. It is however not necessary that all output sets are used for the inference mechanism. The crisp output value is calculated using the Centroid defuzzification operator. The described Fuzzy Logic System (FLS) is defined in [7] as a Standard Additive Model (SAM). Following the same reasoning as in Kosko [8], and with assumptions 1, 2 and 4 it is possible to write the FLS as:

$$FLS(x) = Centroid\left(\sum_{j=1}^r \mu_j(x)B_j(y)\right) = \frac{\int_{\mathbb{R}} y \sum_{j=1}^r \mu_j(x)B_j(y) dy}{\int_{\mathbb{R}} \sum_{j=1}^r \mu_j(x)B_j(y) dy} = \frac{\sum_{j=1}^r \mu_j(x)b_j c_j^B}{\sum_{j=1}^r \mu_j(x)b_j} = \frac{\sum_{j=1}^r \mu_j(x)c_j^B}{\sum_{j=1}^r \mu_j(x)} = \sum_{j=1}^r \mu_j(x)c_j^B. \quad (6)$$

This equation describes the fuzzy relationship from a given input x to a given output y . This relationship is proportional, although for controllers sometimes an integrating relationship is preferred. In the sequel a more general SISO representation of the FL system is used, based on the concatenation of a linear system and a FL system as shown in Fig. 2.

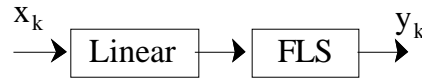


Fig. 2 General SISO Fuzzy System

The representation of this general Fuzzy System is then written in state space form as:

$$\begin{cases} s_{k+1} = Es_k + Fx_k \\ y_k = FLS(Gs_k + Hx_k) = \sum_{j=1}^r \mu_j(Gs_k + Hx_k)c_j^B. \end{cases} \quad (7)$$

It is clear that equation (7) reduces to (6) by setting the matrices E , F and G to zero and $H = 1$. Remark that with this notation it is possible to handle inputs x_k that have more than one elements. Yet, the fuzzy system itself remains SISO.

B. NL_q systems

Suykens [14] defines an NL_q system as a concatenation of q nonlinear and linear subsystems, denoted in state space form as

$$\begin{cases} p_{k+1} = \Gamma_1(V_1\Gamma_2(V_2\dots\Gamma_q(V_qp_k + B_qw_k))\dots + B_1w_k) \\ e_k = \Lambda_1(W_1\Lambda_2(W_2\dots\lambda_q(W_qp_k + D_qw_k))\dots + D_1w_k) \end{cases} \quad (8)$$

and which relates to a recurrent network of the form

$$\begin{cases} p_{k+1} = \sigma_1(V_1\sigma_2(V_2\dots\sigma_q(V_qp_k + B_qw_k))\dots + B_1w_k) \\ e_k = \sigma_1(W_1\sigma_2(W_2\dots\sigma_q(W_qp_k + D_qw_k))\dots + D_1w_k). \end{cases} \quad (9)$$

The V_i and W_i matrices denote the linear part of the NL_q system. σ_i is a sector bounded linear or nonlinear function with the property

$$0 \leq \frac{\sigma_i(\omega)}{\omega} \leq 1. \quad (10)$$

σ_i applied on a vector or matrix is taken elementwise. Γ and Λ are diagonal matrices with diagonal elements $\sigma_i(\omega_i)/\omega_i$ such that $\|\Gamma_i\|_l \leq 1$ and $\|\Lambda_i\|_l \leq 1$ with $\|\cdot\|_l$ the 1, 2 or infinity norm. p_k is the state space parameter of the NL_q system and w_k the exogeneous input. Useable functions for σ are the linear function $lin(\omega) = \omega$, the $\tanh(\omega)$ function used in many neural networks and the $sat(\omega)$ function, defined as

$$sat(\omega) \begin{cases} = \omega & |\omega| \leq 1 \\ = 1 & \omega > 1 \\ = -1 & \omega < -1 \end{cases} \quad \begin{array}{c} \text{sat}(\omega) \\ \uparrow \\ \begin{array}{c} 1 \\ \hline \hline \hline 0 \\ \hline \hline \hline -1 \end{array} \\ \omega \end{array} \quad (11)$$

C. Fuzzy Systems as NLq's

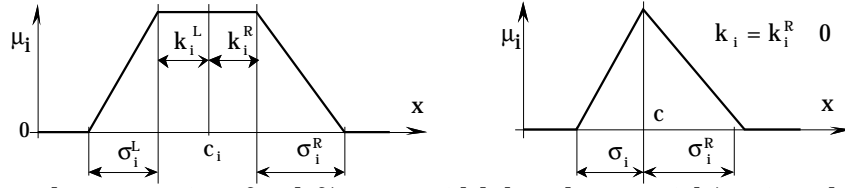


Fig. 3 General representation of left) a trapezoidal shaped set right) a triangular shaped set

Based on the $sat(\omega)$ function it is possible to define a fuzzy trapezoidal set as given in Fig. 3. These sets can be formulated as

$$\mu_i(x) = \frac{1}{2}sat\left(2\frac{x-c_i+k_i^L}{\sigma_i^L}+1\right) + \frac{1}{2}sat\left(-2\frac{x-c_i-k_i^R}{\sigma_i^R}+1\right). \quad (12)$$

For triangular shaped sets k_i^L and k_i^R are zero. Define the positive definite shift vectors and slope vectors

$$K^R = [k_1^R, k_2^R, \dots, k_r^R]^T \quad K^L = [k_1^L, k_2^L, \dots, k_r^L]^T \quad S^L = \left[\frac{1}{\sigma_1^L}, \frac{1}{\sigma_2^L}, \dots, \frac{1}{\sigma_r^L}\right]^T \quad S^R = \left[\frac{1}{\sigma_1^R}, \frac{1}{\sigma_2^R}, \dots, \frac{1}{\sigma_r^R}\right]^T. \quad (13)$$

Define the center vectors

$$c^A = [c_1^A, c_2^A, \dots, c_r^A]^T \quad c^B = [c_1^B, c_2^B, \dots, c_r^B]^T \quad (14)$$

in which the centers for the fuzzy input sets c_i^A are defined as in figure 3, while the centers for the output sets c_i^B are defined in (5). The indices $1, 2, \dots, r$ correspond with the r rules used for the fuzzy inference. It is not necessary that the rules are ordered in any way.

Lemma 1 The Fuzzy Logic System (7) can be written as an NL_2 system.

Proof: using straightforward calculation, it can be shown that equation (12) can be written in the form

$$\Phi(x) = \frac{1}{2} \begin{bmatrix} I_r & I_r \end{bmatrix} sat \left(2 \begin{bmatrix} S^L \\ -S^R \end{bmatrix} x + 1 + 2 \begin{bmatrix} (K^L - c^A) \odot S^L \\ (K^R + c^A) \odot S^R \end{bmatrix} \right) \quad (15)$$

with $\Phi = [\mu_1, \mu_2, \dots, \mu_r]^T$ and I_r an identical matrix of size r . The operator \odot denotes the Hadamar-Shur product (also written as ‘*’ in different mathematical programs). Under assumption 1 equation (7) becomes

$$\begin{cases} s_{k+1} = Es_k + Fx_k \\ y_k = (c^B)^T \Phi(Gs_k + Hx_k) \end{cases} \quad \text{and thus} \quad \begin{cases} s_{k+1} = \text{lin}(V_1 sat(V_2 s_k + B_2' x_k + B_2'')) \\ y_k = \text{lin}(W_1 sat(W_2 s_k + D_2' x_k + D_2'')) \end{cases} \quad (16)$$

with

$$B_2' = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad B_2'' = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad D_2' = \begin{bmatrix} 2S^L H \\ -2S^R H \end{bmatrix} \quad D_2'' = \begin{bmatrix} 2(K^L - c^A) \odot S^L + 1 \\ 2(K^R + c^A) \odot S^R + 1 \end{bmatrix} \quad (17)$$

which corresponds to the NL_2 system

$$\begin{cases} s_{k+1} = \Gamma_1(V_1 \Gamma_2(V_2 s_k + B_2 w_k)) \\ y_k = \Lambda_1(W_1 \Lambda_2(W_2 s_k + D_2 w_k)) \end{cases} \quad (18)$$

with $p_k = s_k$, $w_k = [x_k \ 1]^T$ and

$$V_1 = [E \ F] \quad V_2 = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad B_1 = 0 \quad B_2 = [B_2' \ B_2''] \quad (19)$$

$$W_1 = (c^B)^T \frac{1}{2} [I_r \ I_r] \quad W_2 = \begin{bmatrix} 2S^L G \\ -2S^R G \end{bmatrix} \quad D_1 = 0 \quad D_2 = [D_2' \ D_2'']. \quad (20)$$

The two sector bounded functions are $\sigma_1 = \text{lin}(\bullet)$ and $\sigma_2 = \text{sat}(\bullet)$. Λ_1 and Γ_1 are two identical matrices, while Λ_2

and Γ_2 are diagonal matrices with diagonal elements $\text{sat}(\omega_i)/\omega_i$ that satisfy property (10). \square

III. STABILITY OF NLQ SYSTEMS

Knowing that the FLS that is depicted in the previous section can be written as an NL_2 system, it is possible to make use of the stability properties useable for NL_q systems.

Theorem 1 A sufficient condition for global asymptotic stability of the autonomous SISO Fuzzy Logic system

$$p_{k+1} = \sigma_1(V_1 \sigma_2(\dots \sigma_{q-1}(V_{q-1} \sigma_q(V_q p_k)) \dots)) \quad (21)$$

with $V_i \in \mathbb{R}^{n_i \times n_{i+1}}$ ($n_1 = n_{q+1} = n$) and σ_i a sector bounded function that is applied elementwise, is that a diagonal matrix Δ can be found such that

$$\|\Delta V_{tot} \Delta^{-1}\|_2 = \beta(\Delta) < 1 \quad \text{with} \quad V_{tot} = \begin{bmatrix} 0 & V_2 & & 0 \\ & 0 & V_3 & \\ & & \dots & \\ & & & 0 & V_q \\ V_1 & & & & 0 \end{bmatrix}. \quad (22)$$

Proof: see [14]. \square

Theorem 2 A sufficient condition for I/O stability of the non-autonomous system (8) is that a diagonal matrix Δ can be found, such that

$$\|\Delta V'_{tot} \Delta^{-1}\|_2 = \beta(\Delta) < 1 \quad \text{with} \quad V'_{tot} = \begin{bmatrix} 0 & V'_2 & & 0 \\ & 0 & V'_3 & \\ & & \dots & \\ & & & 0 & V'_q \\ V'_1 & & & & 0 \end{bmatrix} \quad \text{and} \quad V'_i = \begin{bmatrix} V_i & B_i \\ 0 & I \end{bmatrix}. \quad (23)$$

Proof: see [14]. \square

Within this paper we only discuss the stability theorems that make use of a diagonal matrix Δ , which guarantees global asymptotic stability for both the autonomous and non-autonomous cases. In practise these theorems are too conservative. This is certainly the case for FL systems in which only two sets are used for any input x . The unused sets remain in their saturated form. Less conservative stability criterions for NL_q systems are available [15] [16] but will not be discussed in the present paper. These criterions make use of matrices of full rank instead of the diagonal matrix Δ and impose local linear demands on the system, which relates to the use of LMI's (see also Hindi and Boyd in [5]).

IV. APPLICATION

Assume a linear system, with a known nonlinearity that can be modelled using a FL system (FLS). The system is controlled with a FL controller (FLC). The closed loop scheme is given in Fig. 4.

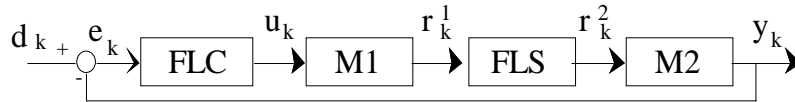


Fig. 4 Fuzzy Logic Control of a combination of linear and FL systems

Lemma 2 The closed loop system of Fig. 4 and given by

$$FLC: \begin{cases} z_{k+1} = E^c z_k + F^c e_k \\ u_k = \text{lin}(W_1^c \text{sat}(W_2^c z_k + D_2^{c'} e_k + D_2^{c''})) \end{cases} \quad M1: \begin{cases} x_{k+1}^1 = A^1 x_k^1 + B^1 u_k \\ r_k^1 = C^1 x_k^1 + D^1 u_k \end{cases} \quad (24)$$

$$FLS: \begin{cases} v_{k+1} = E^s v_k + F^s r_k^1 \\ r_k^2 = \text{lin}(W_1^s \text{sat}(W_2^s v_k + D_2^{s''})) \end{cases} \quad M2: \begin{cases} x_{k+1}^2 = A^2 x_k^2 + B^2 r_k^2 \\ y_k = C^2 x_k^2 + D^2 r_k^2 \end{cases} \quad (25)$$

is an NL_3 system.

Proof: Define the closed loop system state space parameter $p_k = [z_k \ x_k^1 \ v_k \ x_k^2]^T$, the intermediate values t_k^1 and t_k^2 , and the exogeneous input $\omega_k = [d_k \ 1]^T$. It is possible to write the closed loop system as

$$t_k^1 = \begin{bmatrix} \text{lin} \left(\begin{bmatrix} I & 0 \\ & I \\ & & I \\ 0 & & & I \end{bmatrix} p_k + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \omega_k \right) \\ \text{sat} \left(\begin{bmatrix} 0 & 0 & W_2^s & 0 \end{bmatrix} p_k + \begin{bmatrix} 0 & D_2^{''s} \end{bmatrix} \omega_k \right) \end{bmatrix} \quad t_k^2 = \begin{bmatrix} \text{lin} \left(\begin{bmatrix} I & 0 \\ & I \\ & & I \\ 0 & & & W_1^s \end{bmatrix} t_k^1 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \omega_k \right) \\ \text{sat} \left(\begin{bmatrix} W_2^c & 0 & 0 & -D_2^{'c} C^2 & -D_2^{'c} D^2 W_1^s \end{bmatrix} t_k^1 + \begin{bmatrix} D_2^{'c} & D_2^{''c} \end{bmatrix} \omega_k \right) \end{bmatrix} \quad (26)$$

such that

$$p_{k+1} = \text{lin} \left(\begin{bmatrix} E^c & 0 & 0 & -F^c C^2 & -F^c D^2 & 0 \\ 0 & A^1 & 0 & 0 & 0 & B^1 W_1^c \\ 0 & F^s C^1 & E^s & 0 & 0 & F^s D^1 W_1^c \\ 0 & 0 & 0 & A^2 & B^2 & 0 \end{bmatrix} t_k^2 + \begin{bmatrix} F^c & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \omega_k \right). \quad (27)$$

The proof follows by choosing

$$V_1 = \begin{bmatrix} E^c & 0 & 0 & -F^c C^2 & -F^c D^2 & 0 \\ 0 & A^1 & 0 & 0 & 0 & B^1 W_1^c \\ 0 & F^s C^1 & E^s & 0 & 0 & F^s D^1 W_1^c \\ 0 & 0 & 0 & A^2 & B^2 & 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} I & & & & 0 \\ & I & & & \\ & & I & & \\ & & & I & \\ 0 & & & & W_1^s \\ W_2^c & 0 & 0 & -D_2^{'c} C^2 & -D_2^{'c} D^2 W_1^s \end{bmatrix} \quad (28)$$

$$V_3 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & W_2^s & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} F^c & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ D_2^{'c} & D_2^{''c} \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & D_2^{''s} \end{bmatrix}. \quad (29)$$

□

Remark that in the case that the FL controller is chosen as a causal system ($H=0$), W_2^s becomes zero and the closed loop reduces to an NL_2 system. The stability of the autonomous and non-autonomous closed loop system can now be analysed with theorems 1 and 2. For the autonomous case this means that we seek a diagonal matrix Δ such that

$$\left\| \Delta \begin{bmatrix} 0 & V_2 & 0 \\ 0 & 0 & V_3 \\ V_1 & 0 & 0 \end{bmatrix} \Delta^{-1} \right\|_2 = \beta(\Delta) < 1, \text{ or } \left\| \Delta \begin{bmatrix} 0 & V'_2 & 0 \\ 0 & 0 & V'_3 \\ V'_1 & 0 & 0 \end{bmatrix} \Delta^{-1} \right\|_2 = \beta(\Delta) < 1 \quad (30)$$

for the non-autonomous case. The reader should note that further simplifications are possible in these equations, but these lay beyond the scope of this paper. Remark also that finding the Δ matrices is a convex problem that can be solved in polynomial time with classic optimization tools that are available in most mathematical programs.

V. CONCLUSIONS

This paper studied the control of dynamic systems when using a Fuzzy Logic controller. A Single Input-Single Output controller is described within the framework of NL_q theory. This paper showed that it is possible to write a Fuzzy Logic System as an NL_q system such that the stability theorems that are proven for NL_q systems can be used. The theory demands that the controlled plant is adequately described using either a linear model, a FL model, or a mixed form of both

(Certainty Equivalence Principle). The FL system that is described in this paper is a specific form of a Mamdani Fuzzy System with triangular and trapezoidal sets.

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