

System Identification - A Frequency Domain Approach

Errata list

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
xxxv	-3, -8	Richardson	Richards
20	(1-32)	$\sum_{k=1}^N e^{2(k, \theta)}$	$\frac{1}{2} \sum_{k=1}^N e^{2(k, \theta)}$
21	(1-33)	$\frac{1}{N} \sum_{k=1}^N e(k, \theta) \dots$	$\sum_{k=1}^N e(k, \theta) \dots$
23	(1-40)	$e^T(\theta) We(\theta)$	$\frac{1}{2} e^T(\theta) We(\theta)$
57	4	$n_u = 0$	$n_u(t) = 0$
63	-14	$\exp(j\text{rand}(255, 1))$	$\exp(2\pi j\text{rand}(255, 1))$
73	2, 10	$E\{ \}$	$\mathcal{E}\{ \}$
120	11	$b = 2\pi k_1 f_0^2$	$b = 2\pi k_1 f_0$
161 (see also Note a)	(5-78c)	$L_r \frac{1}{\sqrt{\pi t}} + \lambda_r e^{\lambda_r^2 t} \text{erfc}(-\lambda_r \sqrt{t})$	$L_r \left(\frac{1}{\sqrt{\pi t}} + \lambda_r e^{\lambda_r^2 t} \text{erfc}(-\lambda_r \sqrt{t}) \right)$
161 (see also Note a)	-3	$\text{Re}(\lambda_r^2) < 0$ or $ \text{Re}(\lambda_r) < \text{Im}(\lambda_r) $	$ \angle \lambda_r > \pi/4$
163	-3	$X_N(s) = X(z) - \tilde{X}(z)$	$X_N(s) = X(s) - \tilde{X}(s)$
171	(5-131)	$S(j2\pi g)$	$S(j2\pi g) e^{j2\pi(f-g)t}$
233	24	$\sum_{m=1}^3 Y^{[m]}(k) / U^{[m]}(k)$	$\frac{1}{3} \sum_{m=1}^3 Y^{[m]}(k) / U^{[m]}(k)$
263	2	$\frac{-\Omega_k^r U(k)}{\sigma_e(\Omega_k, \theta)} + \dots$	$\frac{-\Omega_k^r U(k)}{\sigma_e(\Omega_k, \theta)} - \dots$
263	(7-220)	$e_{\text{re}}(\theta^{i-1}, Z)$	$\varepsilon_{\text{re}}(\theta^{i-1}, Z)$
263	(7-221)	$e_{\text{re}}(\theta^{i-1}, Z)$	$\varepsilon_{\text{re}}(\theta^{i-1}, Z)$

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
284	(8-10)	$\hat{\alpha}(\Omega_k, \theta, \hat{Z}(k))$	$ \hat{\alpha}(\Omega_k, \theta, \hat{Z}(k)) ^2$
287	(8-21)	$\mathcal{E}\{ \hat{\alpha}(\Omega_k, \hat{\theta}_{ML}(Z_0), \hat{Z}(k)) ^2\}$	$(\mathcal{E}\{ \hat{\alpha}(\Omega_k, \hat{\theta}_{ML}(Z_0), \hat{Z}(k)) ^2\})^2$
308	(8-66)	$V_{ML}{}^T(\hat{\alpha}(Z_0))$	$V_{SML}{}^T(\hat{\alpha}(Z_0))$
310	(8-79)	$\mathcal{E}\{ \hat{\alpha}(\Omega_k, \hat{\theta}_{ML}(Z_0), \hat{Z}(k)) ^2\}$	$(\mathcal{E}\{ \hat{\alpha}(\Omega_k, \hat{\theta}_{ML}(Z_0), \hat{Z}(k)) ^2\})^2$
316 (see also Note b)	(8-106)	$\dots = Ff_0 = 0$	$\dots = O(F \lambda ^F)$ with $ \lambda < 1$
317 (see also Note b)	(8-113)	$\dots = Ff_0 = 0$	$\dots = O(F \lambda ^F)$ with $ \lambda < 1$
317	-9	(8-109)	(8-108)
318	-13	very	every
328	(9-10)	$\dots F_{i_{22}}^{-1} \dots$	$\dots (F_{i_{22}} - F_{i_{21}}F_{i_{11}}^{-1}F_{i_{12}})^{-1} \dots$
354	9 and (10-3)	$g_d(k)$	$g_{ZOH}(k)$
370	(10-17)	$N\lambda$	$N\ln(\lambda)$
375	(10-22)	$\frac{\exp(-\sum_{k=0}^{N-1} \frac{ Y(k) - G(z_k^{-1}, \theta)U(k) ^2}{\lambda H(z_k^{-1}, \theta) ^2})}{\pi^N \lambda^N \prod_{k=0}^{N-1} H(z_k^{-1}, \theta) ^2}$	$\frac{\exp(-\sum_{k=0}^{N-1} \frac{ Y(k) - G(z_k^{-1}, \theta)U(k) ^2}{2\lambda H(z_k^{-1}, \theta) ^2})}{2\pi^{N/2} \lambda^{N/2} \prod_{k=0}^{N-1} H(z_k^{-1}, \theta) }$
375 (see also Note b)	(10-25)	$\dots = Nf_0 = 0$	$\dots = O(\lambda ^N)$ with $ \lambda < 1$
391	(11-9)	$T_c(z_k^{-1})$	$I_c(z_k^{-1})$
426	-17	$r = 1, 2, \dots, k-1$	$r = 1, 2, \dots, s-1$
428	-8	$q_0(x) = 1, q_1(x) = x$ and	$q_0(x) = 1$ and
438	-6	$x_{re} \in \mathbb{C}^{2n}$	$x_{re} \in \mathbb{R}^{2n}$
449	-10	$\lim_{N \rightarrow \infty} E\{x^2(N)\} = 0$	$\lim_{N \rightarrow \infty} x(N) = 0$
472 (Corollary 14.30)	12	Lemma 14.31	Theorem 14.29

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
542	-5	$\check{\alpha}(z_0)$	$\check{\alpha}(z_0)$
585	-10	... for frequency domain identification.	... using periodic input signals.
585	17	Levi	Levy
589	-21	Söderström, T., and B. Carlsson (2000)	Söderström, T., and M. Mossberg (2000)

Note a: stability analysis of the impulse response in the \sqrt{s} -domain (eq. (5-78c) p. 161)

The asymptotic behavior ($t \rightarrow \infty$) of

$$f(t) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t} \operatorname{erfc}(-\lambda \sqrt{t}) \quad (\text{E-1})$$

with $\operatorname{erfc}(z)$ the complementary error function, is analyzed as a function of λ . Using $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ and $\operatorname{erf}(-z) = -\operatorname{erf}(z)$, with $\operatorname{erf}(z)$ the error function, (E-1) can be rewritten as

$$f(t) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t} (1 + \operatorname{erf}(\lambda \sqrt{t})) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t} (2 - \operatorname{erfc}(\lambda \sqrt{t})) \quad (\text{E-2})$$

Applying the following asymptotic expansions ($z \rightarrow \infty$) of $\operatorname{erfc}(z)$

$$\begin{aligned} \operatorname{erfc}(-z) &= \frac{e^{-z^2}}{z\sqrt{\pi}} \left(1 + \sum_{m=1}^{\infty} (-1)^m \frac{(2m-1)!!}{(2z^2)^m} \right) & |\angle z| > \pi/4 \\ \operatorname{erfc}(z) &= \frac{e^{-z^2}}{z\sqrt{\pi}} \left(1 + \sum_{m=1}^{\infty} (-1)^m \frac{(2m-1)!!}{(2z^2)^m} \right) & |\angle z| < 3\pi/4 \end{aligned} \quad (\text{E-3})$$

(see Abramowitz and Stegun, 1970) to (E-1) and (E-2) gives

$$f(t) = \begin{cases} 2\lambda e^{\lambda^2 t} + O(t^{-3/2}) & |\angle \lambda| \leq \pi/4 \\ \frac{1}{2\lambda^2 \sqrt{\pi t^{3/2}}} + O(t^{-5/2}) & |\angle \lambda| > \pi/4 \end{cases} \quad (\text{E-4})$$

It shows that $f(t)$ (i) grows exponentially for $|\angle \lambda| < \pi/4$ ($\Leftrightarrow \operatorname{Re}(\lambda^2) > 0$), (ii) decreases

algebraically to zero as $O(t^{-3/2})$ for $|\angle\lambda| > \pi/4$, and (iii) converges to a periodic solution for $|\angle\lambda| = \pi/4$ ($\Rightarrow \text{Re}(\lambda^2) = 0$) as $O(t^{-3/2})$. With the property $\text{erf}(\bar{z}) = \overline{\text{erf}(z)}$ the results for the complex conjugate root $\bar{\lambda}$ can easily be derived from equations (E-1) to (E-4).

References

Abramowitz M. and I.A. Stegun (1970). *Handbook of Mathematical Functions*. Dover Publications, New York (USA).

Note b: sum over the unit circle of functions that are analytic outside the unit circle and zero at infinity (eq. (8-106) p. 316, eq. (8-113) p. 317, and eq. (10-25) p. 375)

Let $F(z^{-1})$ be a rational function of z^{-1} that is analytic outside the unit circle ($|z| \geq 1$) and zero at $z = \infty$ ($F(0) = 0$). The Taylor series of $F(z^{-1})$ w.r.t. z^{-1} at $z^{-1} = 0$ equals

$$F(z^{-1}) = \sum_{r=1}^{\infty} f_r z^{-r} \text{ for any } |z| \geq 1 \quad (\text{E-5})$$

Using

$$\sum_{k=0}^{N-1} z_k^{-r} = \begin{cases} 0 & \text{for } r \neq nN \\ N & \text{for } r = nN \end{cases} \text{ with } n = 0, 1, \dots \quad (\text{E-6})$$

the sum of $F(z^{-1})$ over a uniform grid on the unit circle ($z_k = \exp(j2\pi k/N)$, $k = 0, 1, \dots, N-1$) can be written as

$$\sum_{k=0}^{N-1} F(z_k^{-1}) = \sum_{r=1}^{\infty} f_r \sum_{k=0}^{N-1} z_k^{-r} = N \sum_{n=1}^{\infty} f_{nN} \quad (\text{E-7})$$

Since $|f_r| \leq K|\lambda|^r$, with $z = \lambda$ the pole of $F(z^{-1})$ closest to the unit circle ($|\lambda| < 1$) and K a constant, the absolute value of (E-7) can be bounded above as

$$\left| \sum_{k=0}^{N-1} F(z_k^{-1}) \right| \leq N \sum_{n=1}^{\infty} K|\lambda|^{nN} \leq O(N|\lambda|^N) \quad (\text{E-8})$$

It shows that contribution of (8-106) to (8-107) and of (8-113) to (8-114) converges ($F \rightarrow \infty$) exponentially to zero at the rate of the dominant pole of $\Delta H(z^{-1}, \theta)/H(z^{-1}, \theta)$ and $F(z^{-1}, z^{-1}, \theta)$ respectively. Hence, all consistency claims remain valid.

Let $H(z^{-1})$ be a stable and inversely stable monic rational form in z^{-1} . The natural logarithm of $H(z^{-1})$ equals

$$\ln(H(z^{-1})) = \sum_r \ln(1 - \beta_r z^{-1}) - \sum_l \ln(1 - \alpha_l z^{-1}) \quad (\text{E-9})$$

with α_l, β_r the poles and zeros of $H(z^{-1})$ satisfying $|\alpha_l| < 1$ and $|\beta_r| < 1$. Using eq. (E-6) and the Taylor series expansion

$$\ln(1 - \lambda z^{-1}) = -\sum_{r=1}^{\infty} (\lambda z^{-1})^r / r, \quad (\text{E-10})$$

the sum of $\ln(1 - \lambda z^{-1})$ over a uniform grid on the unit circle ($z_k = \exp(j2\pi k/N)$, $k = 0, 1, \dots, N-1$) can be written as

$$\sum_{k=0}^{N-1} \ln(1 - \lambda z_k^{-1}) = -\sum_{r=1}^{\infty} \frac{\lambda^r}{r} \sum_{k=0}^{N-1} z_k^{-r} = -N \sum_{n=1}^{\infty} \frac{\lambda^{nN}}{nN} \quad (\text{E-11})$$

The absolute value of (E-11) can be bounded above as

$$\left| \sum_{k=0}^{N-1} \ln(1 - \lambda z_k^{-1}) \right| \leq \sum_{n=1}^{\infty} \frac{|\lambda|^{nN}}{n} \leq O(|\lambda|^N) \quad (\text{E-12})$$

Collecting (E-9) and (E-12) gives

$$\left| \sum_{k=0}^{N-1} \ln(H(z_k^{-1})) \right| \leq O(|\lambda|^N) \quad (\text{E-13})$$

where $z = \lambda$ is the pole or zero of $H(z^{-1})$ closest to the unit circle ($|\lambda| < 1$). It shows that (10-25) converges exponentially to zero at the rate of the dominant pole of $\ln(H(z^{-1}))$. Hence, the asymptotic ($N \rightarrow \infty$) equivalence between the time and frequency domain maximum likelihood cost functions remains valid.

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System Identification - A Frequency Domain Approach

Errors in figures

page 62, Figure 2-27 should be replaced by

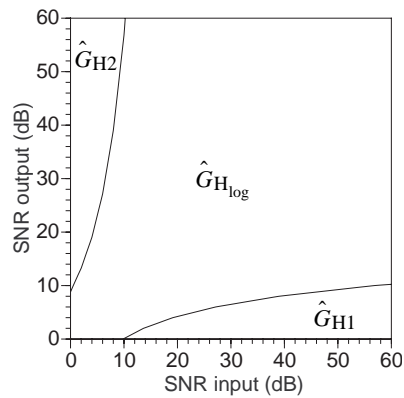


Figure 2-27. Selection between $G_{H_{log}}(j\omega_k)$, $\hat{G}_{H1}(j\omega_k)$, and $\hat{G}_{H2}(j\omega_k)$ as a function of the SNR in case the repeated measurements are not well synchronized.

page 143, Figure 5-5 should be replaced by

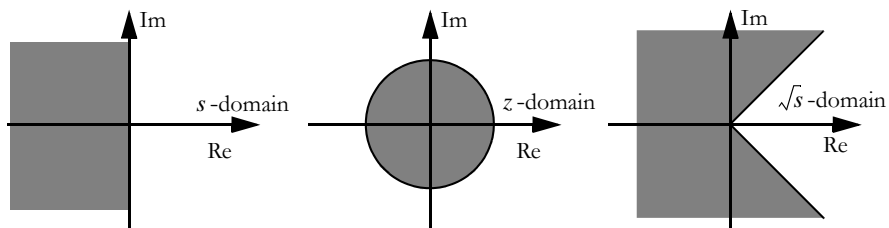


FIGURE 5-5. Grey area: stable and minimum phase regions of, respectively, the poles and zeros. s -domain: $\text{Re}(s) < 0$, z -domain: $|z| < 1$, and \sqrt{s} -domain: $|\angle \sqrt{s}| > \pi/4$.

Errors in figures - continued

page 338, Figure 9-8 should be replaced by

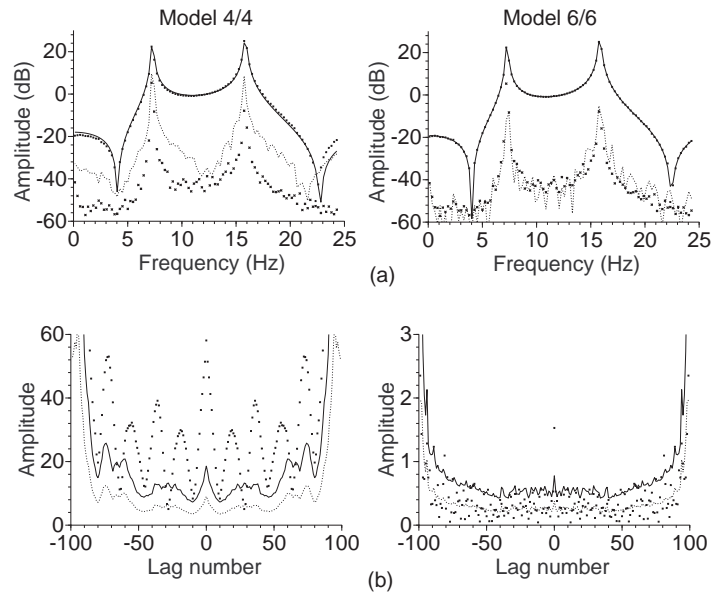


Figure 9-8. Illustration of model error detection and qualification on a vibrating robot arm. (a) the identified transfer function: dots: measurements, — model, --- model errors, x measurement uncertainty σ_G ; (b) $R_{\varepsilon\varepsilon}(m)$, dots: measurement, --- 50% bound, — 95% bound.